



A message from the Executive Director, Tom Young

Our 37th State Tournament wasn't without its requisite obstacles, but all in all, it was a success! My thanks to the students and coaches and support personnel who made it all possible. Special thanks go to Dana Koletar (Associate Director), Stacy Paleen (League President), Tom Kilkelly (HPWT), Mike Reiners (Math Bowl Writer and Emcee), and Gary Kannel (Web Master extraordinaire!). See a recap later in the newsletter.

Now that the State Tournament and Regular season are behind us, we start looking towards the new All State Math Team selection process and ARML competition, Summer Math Institute, and Summer Coaches Conference. Encourage your students to sign up for SMI! **More detailed info on SMI later in the newsletter.**

Please take the survey regarding the Coaches Conference found at:

[Coaches Conference Survey](#)

More information regarding the coaches conference, and the announcement of our 2020 Hall of Fame Class, can be found later in the newsletter.

Please take the survey regarding student fees and the possible need for a scholarship fund to help students who otherwise wouldn't participate in Math League. This is information needed to explore starting such a scholarship funded by the computer software company, Jamf. Details later in the newsletter. Link there and here

[Survey for scholarship](#)

Check out the fifth entry in our continuing series highlighting previous Math Leaguers. **If you know of former students that would like to share their experiences, pass their names along.**

And don't forget to tell your students about the opportunity to participate in the ZIML Online Competition for free for the rest of the school year! And there is a new summer research opportunity!

GO MATH TEAM!

A message from Tom Kilkelly, Head of the Problem Writing Team

This year, for simplicity in grading, we required that all answers would be integers. This proved a challenging and limiting constraint for the problem writers.

But one positive result was that there were far fewer challenges this year than in any past year.

One problem at the state tournament that had several challenges was problem 5 in the Team Event:

5. Find the **sum** of all values for a and b which make the following two systems equivalent, i.e. have the same solution set.

$$\left\{ \begin{array}{l} ax + 2y = b + 1 \\ x + y = 3 \end{array} \right\} \quad \left\{ \begin{array}{l} 2x + y = a^2 + 2 \\ x + 3y = 3 \end{array} \right\}$$

Quick-thinking students immediately saw that $x = 3$ and $y = 0$ was a solution to the system.

$$\text{Then } 6 = a^2 + 2 \rightarrow a = \pm 2 \text{ and therefore, } \pm 6 = b + 1 \rightarrow b = 5 \text{ or } b = -7.$$

A 30-second solution to a Tournament Team Event question?! There must be more to this problem!! What happens if $a = 2$ in the first system? A trick question? Maybe, if it had been on an Individual Event.

I want you to consider a problem that keeps popping up *year* after *year*:

$$\begin{array}{l} \text{If} \quad 2020 = 506^2 - 504^2 \\ \text{and} \quad 2021 = 1011^2 - 1010^2 \end{array}$$

$$\text{Q:} \quad 2022 = m^2 - n^2, \text{ where } m \text{ and } n \text{ are positive integers. What is the value of } m + n \text{ ?}$$

Unfortunately, we won't be able to use this problem next year!

Why not?

Every odd year and every even year which is a multiple of 4 can be written as the difference of two squares: (Proofs: $N = 2k + 1 = (k + 1)^2 - k^2$ and $N = 4k = (k + 1)^2 - (k - 1)^2$)

But no other even years can be written as the difference of two squares. *Can you prove this?*

For further investigation:

$$\begin{array}{l} 2020 = 506^2 - 504^2 = 106^2 - 96^2 \\ 2021 = 1011^2 - 1010^2 = 45^2 - 2^2. \end{array}$$

$$\begin{array}{l} \text{But} \quad 1848 = 463^2 - 461^2 = 233^2 - 229^2 = 157^2 - 151^2 = 83^2 - 71^2 = 73^2 - 59^2 = 53^2 - 31^2 \\ \quad \quad = 47^2 - 19^2 = 43^2 - 1^2. \end{array}$$

**In general, in how many ways can an integer be written as the difference of two squares?
*Can you discover the formula?***

Congratulations to these retirees!!

Daniel Ethier



- 30 years of teaching, 29th year at Mounds Park Academy
- Honors Pre-Algebra, Honors Algebra, and Honors Geometry
- Math League since 2010
- MATHCOUNTS for 28 years
- Cross-Country running for 24 years

“When I started our Math League team, budgets were tight, so I told our upper school director that I would do it for no stipend so long as we had buses to our meets. And there was a meet at Holy Angels (our longest drive) during a snowstorm that season. So that was a good call. But we did have to get to one meet without a bus a few years later, when our AD canceled our bus because he only knew about the three buses for the basketball teams. That was also during a snowstorm. But we made it to the meet in two cars without incident.

In those early days, we often had to scramble to get 8 students to make up a full team. We did more than a few meets with 7. But we’ve had more participants these past few years, and this year, we often had 20 students participating. Some of these are students I taught in 6th grade through 8th grade. A few are also runners on my cross-country team. Getting to know these students so well is definitely one of the joys of doing this.”

Dana Koletar



Dana has been our Associate Director for the past 11 years.

We will miss her insights, ideas, organization, and willingness to work hard to make the League better and better each year.

Our endless thanks!

Enjoy your retirement!

A tribute to Dana from former Associate Director, Shari Colvin:

“I first met Dana many years ago though the Minnesota Council for the Gifted and Talented where we were parent advocates for the appropriate educational opportunities for gifted children. While I was the associate director of the League, Dana became involved in the League as a “Parent Coach” so her son’s school could participate in the League. With the League wanting to move to more of an online platform, Dana was the perfect person to become the associate director with her background experiences as an advocate for students, a parent of a league participant, a league coach, and a degree in computer science.”

The Impact of Math Team

The call went out last summer to Math League alumni to Share Your Story. Here is one alumnus who shared:

Arun Prakash

1996 Graduate of St. Cloud Tech HS

Undergraduate: 1999 Illinois Institute of Technology
BS in Aerospace Engineering

Graduate Degree: 2001 Stanford University
MS in Aeronautics and Astronautics
2007 MIT Sloan School of Management
MBA

Currently works as : CFO Cerebri AI
A pioneer in artificial intelligence and machine learning



The impact of Math Team on my life and learning:

Since graduating from Stanford, I've had a variety of career opportunities. Currently I work as CFO of Cerebri AI, a pioneering company in artificial intelligence and machine learning. Over the years, I have worked on software algorithms at Boeing's Satellite division, been a private equity investor in software and IT services, and a business entrepreneur. Of interest and note: while at MIT, I helped start Terrafugia, a company dedicated to making flying cars more prominent in the transportation sector. I'm also an investor with Space Angels, a company that funds fledgling companies in the space technology sector.

I have always enjoyed math; Math League made that real and fun and competitive. Even as CFO, I am more effective because I understand the math. During mid-90's, non-sports activities were not as recognized, but I lettered in Math League and Knowledge Bowl, so we were recognized formally. It helped when I and my team performed well at the region and state level, because our classmates got behind us. I owe a lot to two great coaches, David Algoe and later Robert Boatz, who left a great legacy at St. Cloud Tech.

My lifelong love of puzzles and brain teasers has been part of some good friendships. My wife is a Stats PhD; I asked her brain teasers very early in our courtship!

Math League encourages students to *learn* math and logic. Successful Math Team students (in fact successful people in general) practice the art of problem-solving and thinking things through versus trying to rely on memorizing facts. They also take the time to go slow enough to get the right answer, testing exceptions and corner cases along the way, and not being too proud to double check their assumptions and calculations. These are the most important legacies of Math League.

2021 Summer Math Institute

June 28th through July 2nd at Augsburg University

The Minnesota State High School Math League is pleased to bring back our Summer Math Institute for 2021!

The dates this year are June 28th-July 2nd.

The Summer Math Institute is two, week-long programs: **one for students entering grades 7-9 in fall of 2021, and the other for students entering grades 10-12 in fall of 2021.**

This year, the programs will not be residential, due to pandemic restrictions.

Given the uncertainty of the pandemic, we will offer our programs either as in-person, daytime only, commuter programs or as online programs. The final determination on the format will be made by June 1st.

If the program is in-person, it will be held on the Augsburg University campus in Minneapolis according to any pandemic restrictions currently in place (i.e, masking, social distancing, etc.)

This year's offerings are:

Grades 7-9: Knots! taught by Annie Perkins

You know prime numbers, but do you know prime knots? Students will explore the relatively new field of knot theory with two mathematical artists. Using algebra, diagrams, string, and other tools, we'll explore both solved and unsolved problems in this branch of topology. A minimum of Algebra 1 and some geometry is necessary background.

Grades 10-12: Probability Theory taught by Ken Suman

Students will study the topic of "Probability Theory", covering all the material necessary to solve the probability theory questions on high school contests like MSHSML, AMC, AIME and ARML. Students should have completed Algebra II and be motivated to learn challenging mathematics not taught in the typical school curriculum.

Cost of program if in-person: \$300 per student

Cost of program if online: \$200 per student

**Apply by April 30th to be given priority consideration.
Late applications will be considered if space is available.**

Direct questions to mathleague@augsborg.edu.

Summer Coaches Conference 2021

Date: Probably August 19 - 20

Last summer, we had to postpone our 40-year celebration due to the pandemic. Hopefully we will be able to hold a celebration this August honoring our new Hall of Famers and toasting to another 40 years!

2020 Hall of Fame Class

(in alphabetical order)

Darryl Anderson: Longtime coach at MACCRAY. Many of his math league students went on to become math teachers. He attended all the summer coaches conferences and was eager to keep on learning. Thanks to Darryl, MACCRAY, a small rural district that joined the league early, has remained in the League even when funds needed to be cut.

Tracy Bibelniaks: Executive Director for 7 years, succeeding Wayne Roberts. Tracy moved the League to its current site, Augsburg University. She reestablished the Summer Math Institute, now in its ninth year.

Dan Butler and Mike Huberty: Longtime coaches at Mounds View HS. Dan and Mike established a culture of success and turned Mounds View into a perennial top 5 school.

Gary Kannel: Longtime coach at Holy Family Catholic HS, Division Coordinator, and League Board member. After many others failed at creating a league scoring website, Gary was able to do what the others couldn't – he created our current scoring website. Not only that, he was able to modify the site to allow students online access to the problems, thus allowing the League to operate during the pandemic.

Dana Koletar: Associate Director for 11 years. Dana was a leader with great insights, ideas, and organization, and a willingness to work hard to make the League better and better each year.

David McMayer: Longtime coach at Southwest HS, Division Coordinator, and League Board member. David is the force behind the Minneapolis division, helping to keep it together through tough years. David coached the All State Math team for 6 years, focusing on expanding the opportunities for our All State members.

Curt Michener: Longtime coach at Elk River HS, Division Coordinator, and League Board member. Curt was instrumental in moving the League to its current section format.

Mary Rueter: Longtime coach at St Cloud Cathedral and St. Johns Prep. Here is an endorsement of Mary from fellow coach Ben Thell

Mary has been a top-notch math team coach for a long time. She has produced outstanding teams at two different schools since I've known her (St. John's Prep and Cathedral). I can't tell you how many times she has brought a team to state or how many team/individual state accomplishments she has. But I do know that it is considerable. Mary has always been instrumental in bringing a high level of competition to the Central Gopher division. Throughout the time I've known her, I've always appreciated the way that I could approach her and pick her brain on how to best help our elite math students realize their potential. Mary is a great coach. Every year at the board of directors meeting and at the summer coaches conference, I've heard the call for nominees for the Hall of Fame or people who should be recognized as they retire. In my opinion, Mary is one of those Hall of Fame coaches.

Jerry Shouts: Longtime coach at Delano HS, Division Coordinator, and League Board member. Participating in math competitions was always popular in Delano. From Math Masters to High School Math League, their rosters were among the largest in the state - and with good coaching, they have always done very well. Jerry Shouts was an integral part of the team of math teachers/coaches in encouraging and celebrating their students' success.

2020 – 2021 State Tournament Results

The MN State HS Mathematics League had its 37th State Tournament on Monday, March 15th. There were over 350 students that participated via a Zoom Meeting. The students represented over 40 high schools.

The meet started off with the Math Bowl. This year's winner was **Linden Lee from Mounds View HS**. Here is one of the problems that Linden and 9 other competitors had to solve:

- A. The year 2021 can be written in the form $F^2 - n^2$, where F is a positive integer whose last digit is 5 and n is some other positive integer. What is the next year with this property? *See answer below.*

The tournament continued with students tackling events A through D. Here is one of the Geometry problems:

- B. On the screen of my graphing calculator point A $(1, \frac{1}{2})$ is 0.2 cm. from the origin. In centimeters, how far from the origin would point B $(5, 10)$ be on this calculator? *See answer below.*

The tournament concluded with the team event. Here is one of the problems:

- C. In a circle centered at O, segments AB and CD are two perpendicular chords. If $AC = 6$ and $BD = 10$, the radius of the circle can be written as \sqrt{b} , where b is square-free. Determine the value of b .

The individual winners of the tournament were:

1 st Linden Lee	10 th grade Mounds View
2 nd Matthew Chen	10 th grade Wayzata
3 rd Jerry Zhang	10 th grade Minnetonka

The all-around winners with the best combined season and tournament scores, who earned a monetary scholarship were:

1 st Matthew Chen	10 th grade Wayzata
2 nd Winston Lu	12 th grade Minnetonka High School
3 rd Linden Lee	10 th grade Mounds View 165.14680 points / 104 points

The winning teams were:

Class AAA

- 1st Wayzata
- 2nd Edina
- 3rd Minnetonka

Class AA

- 1st Mankato West
- 2nd Mounds Park Academy
- 2nd Mahtomedi

Class A

- 1st Holy Family Catholic
- 2nd Breck
- 3rd St. Croix Lutheran

Congratulations to these winners and to all the teams and students who participated in the Tournament!

All State Team Summary

From January to March of 2021, Minnesota All-State Math Team (MN Math Team) mathletes prepared for and competed in three national mathematics events, sponsored by the students at Carnegie Mellon University (CMIMC Math 2021), Harvard University / Massachusetts Institute of Technology (HMMT Spring 2021), and Princeton University (PUMaC 2020*). Due to circumstances, each tournament was held online, which may not have been as fun as travelling to campus, but it allowed for dozens of Minnesota mathletes to experience life in the mathematical fast lane.

CMIMC Math 2021

[CMIMC Math 2021](#) was held 27 February 2021 and comprised two team events (Theoretical Computer Science or TCS, and Team) and three individual events (Algebra / Number Theory, Combinatorics, and Geometry). In individual events, mathletes self-selected to compete in Division 1 (best of the best), or Division 2 (highly competitive). The MN Math Team entered 4.5 six-person teams (MN Gold, Maroon, White, and Orange, with 27 mathletes - three merged with Massachusetts students to form the full team Future Hendrix Fan Club). The weekend also included fun mini-events (bullet chess, escape room, function guessing, trivia), educational talks (college life, entrepreneurship), and an Awards Ceremony.

[Team honors](#): The MN Gold team of Matthew Chen, Evan Erickson, Linden Lee, Jason Wang, Kevin Yang, and David Zhang placed in TCS (48th), Team (28th), and the overall (which includes individual events) Sweepstakes (27th).

[Individual honors](#) (Division 1): Matthew Chen (48th, Geometry), Kevin Yang (29th, Combinatorics)

[Individual honors](#) (Division 2): Michelle Cao (17th, Geometry), Eric Chen (17th, Geometry), Nathan Mihm (Geometry, 5th), Justin Luan (38th, Combinatorics), Minkyu (David) Jun (48th, Algebra Number Theory), Ashley Maki (15th, Combinatorics), Anvaya Shiney-Ajay (17th, Geometry), Joshua Taylor (43rd, Combinatorics), and Leo Witzke (17th, Algebra / Number Theory).

HMMT Spring 2021

[HMMT Spring 2021](#) was held 6 March 2021 and comprised two team events (Team and Guts) and three individual events (Algebra / Number Theory, Combinatorics, and Geometry). The MN Math Team had two 8-person teams accepted, MN Gold and MN White. The weekend included fun mini-events (many, also with an integration bee), interesting talks (Po-Shen Loh keynote on [Applied Pure Math](#) and nine more), and an Awards Ceremony.

[Team honors](#): MN Gold team of Eric Chen, Matthew Chen, Evan Erickson, Sasha Hydrie, Linden Lee, Winston Lu, Aurora Wang, and Kevin Yang placed in Team, 15th; Guts, 22nd; Overall Sweepstakes, 26th; and Mathlete Aggregate, 28th. The MN White team of Michelle Cao, Garv Khurana, Jonah Kramer, Nathan Mihm, Luke Patefield, Henry Scheible, Leo Witzke and Bob Zhang (Overall Sweepstakes, 60th).

[Individual honors](#): Matthew Chen (Geometry, 28th), Linden Lee (Algebra / Number Theory, 15th; Combinatorics, 33rd; Overall, 37th). By virtue of his Overall Individual Top-50 finish, Linden Lee has advanced to the HMMT Finals in late April.

PUMaC 2020*

[PUMaC 2020*](#) (delayed to 2021, hence *) was held 27 March 2021 and comprised two team events (Team Test, and a prior week-long Power Round) and two individual events (mathletes each chose two from Algebra, Combinatorics, Geometry, and Number Theory). The MN Math Team had two 8-person teams, MN Gold and MN Maroon. The weekend also included fun mini-events and an Awards Ceremony.

[Honors](#): As we go to press, the results are not yet formally published; however, it has been announced that the MN Gold Team of Kenneth Chen, Matthew Chen, Evan Erickson, Linden Lee, Minkai Li, Jason Wang, Kevin Yang, and David Zhang placed 9th in the Team Test.

Minnesota All-State Math Team 2021-22

Invitations to the 2021-22 MN Math Team spring practices will be extended soon to select individuals, based primarily upon Math League results and AMC scores. The next event is the online American Regions Mathematics League ([ARML](#)) event 5 June 2021.

Free Texts from the Summer Math Institute!!

Dr. Ken Suman, a retired mathematics professor at Winona State University, has been our lead teacher in the 10th – 12th grade SMI for the past two years. In 2018, students at SMI studied Counting Techniques and in 2019, they studied the Theory of Equations.

Dr. Suman wrote texts for the classes, specifically with Math League in mind. These texts are a goldmine of information. Dr. Suman has willingly shared his expertise and suggested that the texts be available to all Math Leaguers.

To that end, the pdfs of the texts can be found at scoringmnmathleague.org under the Coaches Corner tab. Then click on Topic Resources and you'll see the links.

Survey for Possible Scholarship Program

As an offshoot of preliminary talks with the computer software company Jamf, we are exploring the idea of setting up a scholarship fund for underserved students. We need more information to share with Jamf to make a better decision as to the need for such a scholarship. Please take the time and fill out the following survey.



[Survey for scholarship](#)



The Roberts Award Scholarship

The Roberts Award Scholarship(s) were established in honor of the League founder, Dr. Wayne Roberts of Macalester College.

The Scholarship(s) are offered to help offset the costs for students interested in attending an out-of-state math opportunity. They are offered once each year. A set amount of funds will be available each year, and multiple awards are possible.

Deadline to apply for this season is April 30, 2021

Applications can be found on our web site at: http://mnmathleague.org/?page_id=1033



**Great Opportunity!!
ZIML Competition News**

Minnesota Math League students can participate in the ZIML Competitions for free for the rest of the school year!! Here are the instructions:

To enroll in the contests, students will need to create a free account on the site ziml.areteem.org, if they don't have one already. Once they are logged in, they can go to the "Monthly Contests" page (<https://ziml.areteem.org/ziml/monthlycontests.php>) and choose which division they want to register for.

Once in the division page they will be asked for a payment method OR an enrollment key; they can use enrollment code MNMLSp2021 in the box below that says "Enroll using an enrollment key", so no payment information is necessary. If a student wishes to participate in more than one division, they need to repeat these steps for each division.

Then, on the weekend of the contest, they need to go back to the contest page and the contest quiz will be available. Students can participate any time during the weekend (Friday at 00:01 to Sunday 23:59), but once they start the quiz the timer will give them one hour to finish.

The code we provided you is valid for all remaining monthly competitions for the current school year. They will need to follow the above steps each time to register.

In addition to the monthly competitions, students can find daily problems of various levels (the Daily Magic Spells), as well as all past AMC 8, 10, 12, and AIME tests for practice.

Zoom International Math League





Great Opportunity!!
Summer Research Program

The Summer STEM Institute (SSI) is a six-week virtual summer research program for high school students. The program consists of three main parts:

Research and data science bootcamp: Students learn how to design and conduct data science research projects. Students learn about the research process and how to apply statistical and machine learning methods to address scientific questions with real-world impact.

Masterclass lecture series: Students are connected to accomplished young adults around the world. Students have the opportunity to learn about entrepreneurship from Forbes 30 Under 30 recipients, science research from International Science and Engineering Fair (ISEF) grand prize winners, and other topics from speakers with diverse backgrounds and experiences.

Mentored research project: Students complete a research project through a rigorous, hands-on learning experience under the guidance of mentors. Previously, students have worked in a variety of fields, from computational biology to quantum physics.

Many students who attended the program last summer said SSI changed their lives and asked for the SSI team to continue running SSI in future years. You can view [distinguished research projects](#) on the program website.

Applications for the 2021 program are open until April 16th.

To learn more about the program and apply, students can visit our website at: <https://www.summersteminstitute.org/>.

Problem Corner

an effort to spur conversation

If you'd like to contribute a problem or send in a solution, email tomyoungmathman@gmail.com

Student solutions encouraged!

Newsletter #24 Puzzler:

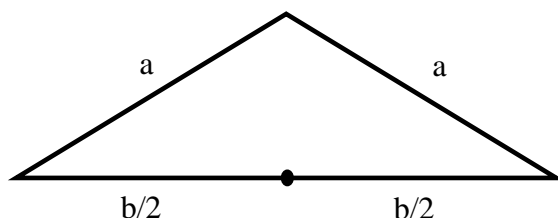
Define the extouch midpoint of a triangle as such: Beginning at a vertex, travel halfway around the triangle. The point reached is an extouch midpoint. There are 3 such extouch midpoints in a triangle.

Problem 1: Prove the 3 extouch midpoints lie on the 3 sides of the triangle, one per side.

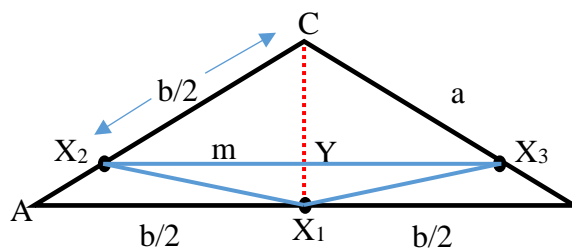
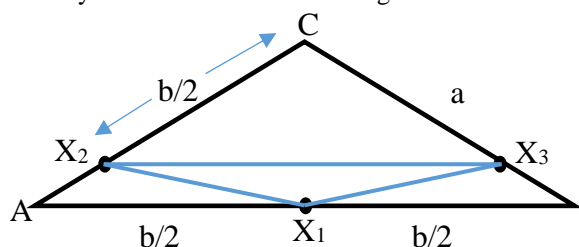
Problem 2: Given an isosceles triangle with legs of length a and base of length b , find the formula (in terms of a and b) for the base of the isosceles triangle formed by joining the extouch midpoints of the original triangle.

Solution Problem 2:

The perimeter is $2a + b$, therefore the semi-perimeter is $a + b/2$. Starting at the apex of the isosceles triangle and traveling the semi-perimeter of $a + b/2$, you land at the midpoint of the base. That is one of the extouch midpoints.



Starting at one of the base vertices, and traveling $a + b/2$ lands you on one of the legs, since $b/2 < a$. Therefore, the two other extouch midpoints are symmetrical and lie on the legs.



Dropping a perpendicular from C, intersecting X_2X_3 at Y, two similar triangles CYX_2 and CX_1A are created. Therefore

$$\frac{AC}{AX_1} = \frac{CX_2}{X_2Y} \quad \text{or} \quad \frac{a}{\frac{b}{2}} = \frac{\frac{b}{2}}{m} \quad \text{or} \quad m = \frac{b^2}{4a} \quad \text{Therefore} \quad X_2X_3 = \frac{b^2}{2a}$$

Newsletter #25 Puzzler (<https://paigefashion.com/wiki/invariant-principle-definition/index.html>)

Part One: Alice writes the numbers 1, 2, 3, 4, 5, and 6 on a blackboard. Bob selects two of these numbers, erases both of them, and writes down their sum on the blackboard. For example, if Bob chose the numbers 3 and 4, the blackboard would contain the numbers 1, 2, 5, 6, and 7. Bob continues until there is only one number left on the board. What are the possible values of that number?

Part Two: Alice writes the numbers 1, 2, 3, 4, 5, and 6 on a blackboard. Bob selects two of these numbers, erases both of them, and writes down their **positive difference** on the blackboard. For example, if Bob chose the numbers 3 and 4, the blackboard would contain the numbers 1, 1, 2, 5, and 6. Bob continues until there is only one number left on the board. What are the possible values of that number?