

What is the “Theory of Equations” all about?

“Theory of Equations” is an old title for the study of polynomials.

It is fair to say that the study of polynomials has two mostly disjoint sides to it: a precalculus side and a theory of equations (TOEs) side. So, we could define TOEs as everything about polynomials not covered in precalculus. But that only tells us what TOEs *isn't*. Can we say what TOEs *is*? And without resorting to lame jokes like TOEs are your lower digits.

A different human body analogy might help. Consider the subjects of anatomy and physiology. Anatomy deals with the form and structure of body parts while physiology deals with how changes in one body part effect other body parts. Using this dichotomy, we can understand precalculus (and to a large extent, calculus too) as the physiology of polynomials and TOEs as the anatomy of polynomials. In precalculus you study polynomials as a basis for modeling change. In TOEs you dissect polynomials to see what form and structure the parts (*i.e.* coefficients) have.

In precalculus you study the shape of graphs of polynomials – especially how the shapes change. For example, where does the graph switch from increasing to decreasing? Where is the graph convex?

In contrast, in TOEs you study the structure of the coefficients of polynomials. For example, do the coefficients exhibit any sort of symmetry, do the coefficients alternate from positive to negative – and why would that be important? Also, what conditions on the coefficients are necessary for a polynomial to factor? And most importantly as it relates to this SMI course, what can the coefficients of a polynomial tell us about the zeroes of a polynomial?

The content historically placed under the heading “theory of equations” has mostly disappeared from high school and college curriculums. This occurred as part of a purposeful shift away from the study of “math for math’s sake” in high school as a way of filtering out students for college admission. However, TOE problems continue to appear with regularity in “contest math” notwithstanding the absence of TOE content in high school and college math courses. Obviously, this is something of a dilemma for students who get involved in math contests.

The purpose of this SMI course is to fill in this missing link. This course will specifically focus on presenting those aspects of TOEs needed to solve the theory of equation questions that appear on our MSHSML tests and on AMC, AIME and ARML tests. This course will include a large problem-solving component.

The necessary math background for success in this course is Algebra II. You will not need a calculator or a laptop for this course. Pencils, paper and a problem-solving mindset will suffice.