

**MINNESOTA STATE
HIGH SCHOOL
MATHEMATICS
LEAGUE**

**LEAGUE MANUAL
2024 -2025**

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I. Introduction to the Minnesota State High School Math League

The Minnesota State High School Mathematics League was founded in 1980-81 by Macalester professor Wayne Roberts and was modeled after leagues that have flourished along the U.S. eastern seaboard since the mid-1940s. The League has grown from four participating schools in its first year to over 160 participating schools today.

The League exists to identify students with exceptional interest and/or mathematical ability, give them recognition and encouragement, bring them together with similarly motivated students, and introduce them to topics not commonly taught in the high school curriculum.

Although the League's focus and related activities tend to draw the attention of students with exceptional ability and interest in mathematics, they are inextricably relevant to educational practices for students at all levels of mathematical aptitude. The League has always believed that a program for gifted students is shortsighted if it is not developed in a way that strengthens mathematics education for all students.

Currently, the Minnesota State High School Mathematics League administers the original annual statewide math competition (known as the Math League) open to all qualified students attending schools in Minnesota and schools on the border of Minnesota. Also, as part of the All State Math Team, the League invites high-performing students to participate in the American Regions Mathematics League (ARML) and other national contests. During the summer, the League sponsors residential math programs (the Summer Mathematics Institute, or SMI) for 7th – 12th grade students.

II. Math League Competition Rules

A. Eligible Individuals and Teams/Registering a team

Math League is a competition for both individuals and teams. Math League has two components: the regular season and the State Tournament. Members are U.S. high schools in or bordering on the State of Minnesota. Member teams are placed in a Division and Section based on criteria outlined below.

Each year, schools need to notify the League of their intent to participate by submitting a letter of intent by September 15th. Each school team is required to pay a fee of \$600 to participate in the League. Payment of the registration fee must be received by October 15. Schools with math team rosters of fewer than 5 students may petition the League by October 15 to request a reduced registration fee.

Individual contestants must be regularly enrolled students in a participating Senior High School or a Junior High School/Middle School in the district of the Senior High School. The number of students that can participate from a school is unlimited.

Students with disabilities may fully participate in the League. Coaches of these students should contact the League office to discuss arrangements for accommodations.

Individual Participation for any student without a Math team

Home-schooled students or students who attend schools having no official Math Team may fully participate with another school's math team unless school policy prohibits such participation. Students may register directly with the League as individuals and will be assigned to the appropriate geographical Division. Students are eligible for all individual awards and honors within that Division and at the state level. Individual students must be accompanied by an adult chaperone at all meets. The cost for an individual student is \$100.

B. Division, Class, and Section Structure

See the Math League website, www.mnmathleague.org, for Division, Class, and Section assignments.

Division Alignment Rules

It is the intention of the board that each Division be composed of at least 5 teams. If a Division drops below 5 teams, it may operate with fewer teams for 1 year. If the Division does not have at least 5 teams the following year, the teams in the Division may be redistributed to existing Division(s).

There are several reasons for wanting, insofar as possible, to have Divisions of at least 5, and more desirably, about 8 teams. They are as follows:

- First and foremost, small Divisions can give rise to a feeling of inequity.
- We face increasing pressure from teams in large divisions to break themselves into smaller Divisions. This exacerbates the problem mentioned above, but it is hard to resist when applicants can point to Divisions already smaller than the ones they propose to create by dividing.

Recognizing the difficulty of always finding 5 schools in a geographic area that want to participate, the League Board passed at its September 2001 meeting the following motion:

“In cases where distance makes it impractical for teams in a Division to come together at one site for each of the five meets of our season, the Division may designate two sites, so long as at least three teams gather at each site. Provisions should be made to have the sites in electronic communication so that teams at each site can see their standing in the meet as each event is graded, and the results should come to the League office as the report of a single Division.”

Division Alignment Policy

New schools, merging of existing schools, schools dropping out of or joining the League, the forming of new athletic conferences: these and other changes require that each fall we do some reorganizing of our divisional structure. In setting up Divisions, the office staff should observe the following guidelines:

- Member schools should, insofar as possible, be put in a division with schools where they are most comfortable.
- Unless prompted by external requests, we should attempt to keep intact the same Divisions from year to year.
- When an existing Division is to be changed slightly by addition or deletion of a team or two, this should be done with the cooperation of the Division Coordinator.
- Great effort should be made to have Divisions consist of at least 5 teams.
- When major realignments are undertaken, coaches of all teams affected should be invited to a meeting to discuss implementation issues. In all cases, alignment of Divisions worked out through negotiations between the office staff and the affected teams must be approved by the League’s Executive Committee.

Division Coordinator

Each Division should, as its last act of business at the conclusion of a season, appoint a Division Coordinator for the following season. It is permissible, even advisable, to have the same person serve as Division Coordinator for several successive years.

The Division Coordinator becomes a member of the League's Board of Directors and represents the Division at the Annual Fall Board Meeting and at any special meetings of the Board. A Division Coordinator who cannot attend a meeting of the Board should appoint another coach from the Division who then becomes a voting member for that meeting. An annual meeting will normally be scheduled in late September. At this meeting, any pending questions about League rules will be settled, alignment of teams into Divisions will be tentatively settled, and host schools in each Division should be designated for the coming season.

Class and Section Structure for State Tournament Qualification

For the purpose of qualifying for the State Tournament, the schools in the League will be organized into three Classes as outlined below. Each Class is composed of 8 Sections with schools placed in a Section based on similar geographic location.

Class Assignment

Placement of teams into Classes shall be based on enrollment numbers from the Minnesota State High School League for the upcoming two-year cycle. Class assignments will be for two-year cycles and are reassigned at the end of the season of odd-numbered years. After a two-year cycle is completed, adjustments in Classes are made based on the averages of the previous two years of performances. Class AA and A teams who averaged in the top 15 overall in the regular season in the previous two years are moved to Class AAA for the following two years. Class A teams who averaged in the top 50 in the regular season in the previous two years are moved to Class AA for the following two years.

Teams that have been moved to a higher Class will be returned to the Class dictated by the most recently available enrollment numbers if the two-year review dictates it. The teams will be moved if, during the two years they were moved to a higher Class, they did not maintain the standard by which they were moved up. For instance, if a Class AA or Class A team had been moved to Class AAA due to being in the top 15 based on their two-year average, but did not maintain that top 15 average, the team would be placed in the appropriate Class for their performance. This could be Class AA, if a Class A team met the performance criteria for placement in Class AA, or moved back to Class A. If a Class A team was moved to Class AA due to being in the top fifty, but did not maintain the top fifty status for the two years they were moved up, the team would be moved back to Class A.

Any school will have the option to “opt up” to a higher Class by declaring their intent to do so prior to October 1 of the beginning of the two-year period. Teams will be notified of approval of their request to opt up by October 15.

Class placement for schools added to the League in the middle of a Class assignment cycle shall be at the discretion of the Executive Committee or its designee.

C. Regular Season Procedures

Overview of Season Structure and Competition

The regular season of the League consists of 5 meets as noted in the schedules. Teams and individuals compete in Divisions. The number of students that can attend a meet is unlimited and each student's score is recorded for individual honors. However, only 8 pre-selected student scores are counted toward the total team score (see further explanation below). Individual and team scores are kept for all meets and cumulative scores for individuals and teams are used to determine invitations to the State Tournament. The League recognizes team accomplishments by awarding trophies in each Division and in each Class.

Meet Rules

Each meet is organized into two parts, “Individual Competition” and “Team Competition,” as follows:

Individual Competition

Each student may compete in all three events during any single meet. The events are labeled A, B, and C but are essentially Algebra topics (event A), Geometry and Trigonometry topics (event B), and Counting, Probability & Statistics, and Number Theory topics (event C). Each event has five questions. All questions are each worth 1 point for a total of 5 points per event. Questions typically increase in difficulty within each event.

In each event, the students are given 15 minutes to solve as many of the five questions as they can. Students are not allowed to communicate with each other during the individual events. Topics for these events vary by meet and are outlined in the appendices. Participants’ scores in individual events count towards their cumulative season individual scoring totals, regardless of whether or not they are one of the pre-selected scoring students (see Team Competition below).

Team Competition

Since the number of participants is unlimited, larger schools would have an advantage if all student scores were counted for the total team score. To mitigate this advantage, coaches must identify 8 students prior to each meet to be the official scoring members of the team for that meet. These 8 scoring students must compete in all three individual events.

The 8 scoring students can vary from meet to meet, but for any given meet, those 8 students representing their school shall be clearly indicated in the Math League Online Scoring and Reporting System roster before the meet begins. Under no circumstances shall a team be allowed to identify its scoring students for a particular meet after the grading of the first event at that meet has begun. It is important to note that the 8 scoring students’ individual scores not only count towards their cumulative season individual scoring total, but also towards the Division-wide, Class, Section and Statewide ranking of their team.

After the individual events are completed, the 8 scoring students (but none of the others who have participated as individuals) compete in a final team event consisting of 6 questions. They are given 30 minutes for this event during which they can cooperate with one another in order to produce and submit a single set of answers. Each question on the team event is worth 5 points, for a total of 30 points. The team event score plus the sums of individual event scores of the 8 team members determines the official team score for the meet. It should be noted that non-scoring-team members may also participate in the team event in a separate space from the scoring team.

Team Composition, Scoring, and other Rules

1) *Age Restrictions on scoring team members*

No more than 6 of the 8 scoring team members shall be beyond the 10th grade.

Penalties for Age Restriction Violations

If all 8 of the scoring team members are beyond the 10th grade then the two students with the highest scores will have their totals removed from the team score. There will be no penalty for the team event.

If 7 of the 8 scoring team members are beyond the 10th grade then the one student in grades 11 or 12 with

the highest score will have their totals removed from the team score. The score for the eligible student in grades 9 or 10 still counts, even if it is the top score. There will be no penalty for the team event.

2) *Tabulation of Points for Individuals*

Individuals (whether on the official scoring team or not) can earn up to 15 points (called the “raw score”) in any one meet for a possible total of 75 points for the five-meet season. The scores of scoring team members will be weighted using the following formula, with the result called the “Power Score”:

$$(\text{Event Score}) = (\text{Base Score}) + 15 \frac{1 + (\text{number of students attempting the question})}{1 + (\text{number of students correctly answering the question})}$$

The Power Scores will be used to rank individuals statewide and within their Division.

Scores are tabulated and the top 50 individuals (including all ties) are posted on the League web page. Cumulative Power Scores are used to determine Divisional honors and individual invitations to the State Tournament.

3) *Tabulation of Points for Teams*

The maximum team score for a meet is 150 points and is based on the raw scores of the scoring team members (15-point perfect score for each of the 8 individual scoring team members and a perfect 30-point team event score) for a possible 750 points for the five-meet season. Cumulative points are tabulated and reported to the League.

4) *Ranking of Teams within Divisions and Sections*

For the sake of ranking within Divisions and Sections, teams are awarded ordinal ranking points at each meet as follows. If n is the number of schools in the Division or Section, then:

- The 1st place team earns $n + 1$ ordinal points
- The 2nd place team earns $n - 1$ ordinal points
- The 3rd place team earns $n - 2$ ordinal points

The pattern continues until the last place team at the meet earns a minimum of one ordinal point.

A team that does not participate in a meet will not receive any points. In case of a tie at a meet, the ordinal points normally awarded for each place are averaged and awarded to each tied team. For example, if there is a 3-way tie for second place, the second, third and fourth place ordinal points are averaged and awarded to each team.

Teams are ranked in their Division and Section by cumulative ordinal points, not overall total season points. If there is a tie in ordinal points after five meets, the Division or Section winner will be the team with the most season points.

5) *Calculator Usage*

The current policy states that **no calculator** can be used on any individual or team event.

Topics for Events

Below is a broad overview of the topic areas of each event. See Appendix C for a complete list of topics covered per event per meet. Keep in mind that topics from previous events can occur in subsequent events.

Event A - Algebra topics

Event B - Geometry and Trigonometry topics

Event C - Counting, Probability & Statistics, and Number Theory topics

Formula Sheet

A League-created formula sheet is to be provided to all students. Students may use this formula sheet for practices, but an unaltered version must be provided for meets. It is suggested that coaches make copies of the formula sheet on colored paper for students to use during meets. The formula sheet will be provided by the League at the state meet. See Appendix D.

Hosting/Running a Meet

FOR MEETS 1 THROUGH 4, the Divisions will have latitude in deciding which of the following options to use.

Option 1

Schools in the Division meet at a common site and conduct meets using paper exams and pencils. Students will be given paper copies of the Events, and they will hand in their solutions once the time is up. Collected papers are then graded and double-checked by the coaches at the site. Scores are recorded on the Math League Online Scoring and Reporting System by the coaches via the Meet Op > Score Entry method.

Option 2

Schools meet individually. Students gather at their own school to compete together in the same place. Students must gather at school. Students competing from various sites (e.g. from home) is not permitted. The coach conducts the meet using the Math League Online Scoring and Reporting System. Students will be given paper copies of the Events, and they will enter their solutions via the Online Scoring System. Scores are verified on the Online Scoring System by the coaches via the Meet Op > Grade Online Taken Events method.

Option 3

Schools meet individually. Students gather at their own school to compete together in the same place. The coach conducts the meets using paper exams and pencils. Students will be given paper copies of the Events, and they will hand in their solutions once the time is up. Collected papers are then graded and double-checked by emailing copies of student papers to another coach at another school. Scores are recorded on the Math League Online Scoring and Reporting System by the coaches via the Meet Op > Score Entry method.

Option 4

Divisions choose a hybrid model where some schools meet in person while other schools compete virtually. The schools that meet at a common site use Option 1. The schools that don't meet at the common site use Option 2 or Option 3.

FOR MEET 5

Divisions are required to meet in person and conduct meets using paper exams and pencils. Students will be given paper copies of the Events, and they will hand in their solutions once the time is up. Collected papers are then graded and double-checked by the coaches at the site. Scores are recorded on the Math League Online Scoring and Reporting System by the coaches via the Meet Op > Score Entry method. Meeting as a Division for Meet 5 helps to further the mission of the League, giving an opportunity for students to meet. It also facilitates distribution of end-of-season awards.

MEET DATES AND TIMES

Divisions or schools will be able to compete on the scheduled Monday or the Tuesday immediately following. Schools competing alone using Option 2 can sign up for time slots for either day. Divisions (Option 1) or schools using Option 3 can hold their meets on Monday or Tuesday also.

To be clear: all members of a team must be physically gathered in the same location to compete at a meet, regardless of the chosen option.

Steps Prior to Meet 1

- Edit team roster adding and removing students
- Student Accounts
 - Each student will have their own account. Go to Team Admin > Student Accounts to generate and retrieve their usernames and passwords.
- At least once, run through the practice Meet with your students so everyone understands the process.

Structure of a Meet where more than one team gathers

Divisions run meets somewhat differently but often employ a structure resembling the following:

- i. Convene all students in a central gathering space
- ii. Welcome students, serve refreshments, and remind students of the rules
- iii. Gather graders in a separate room and review problems and solutions
- iv. Verify that all coaches have predetermined their scoring team students
- v. Announce Event A and direct Event A participants to designated room(s)
- vi. Administer Event A and set up Event B
- vii. Bring Event A student answers to graders
- viii. Post Event A answers so students can learn solutions or challenge scoring
- ix. Repeat previous four steps (iv. – vii.) for Events B and C.
- x. Allow students time to review all solutions to Events A, B, and C or challenge scoring
- xi. Announce team event and move teams to separate rooms
- xii. Administer team event
- xiii. Bring answers to graders
- xiv. Post team solutions and allow challenges
- xv. Disallow challenges after 15 minutes; finalize and report scores

Proctoring

Individual Events

It is suggested that in Individual Events, tests, scratch paper and the formula sheet be laid on desks, face down, before contestants enter the room. It is also suggested that contestants from the same school should not sit next to each other. In some Divisions, contestants write their names and schools on the backs of the exams before turning them over on the signal to start. The proctor should give a two-minute warning before the end of the event. Contestants should lay their pencils down and turn their papers over when time is called. The proctor collects papers off the desks after contestants leave.

Team Event:

Each team participating in the team event requires a proctor. An individual proctor may be assigned more than one team to proctor.

Coaches' Duties

Before the day of the meet, the Host Coach shall prepare an assignment sheet to be given to each visiting coach upon arrival. This sheet should assign coaches (and perhaps extra people as needed from the host school) to:

- Serve as proctors of Events A & C
- Serve as proctors of Event B (This allows proctors for Event B to set up the room for Event B while A is underway, etc.)
- Serve as graders (2, preferably 3 or more)
- Serve as proctors of teams during team event.
- Monitor waiting areas, supervise distribution of refreshments (if any)

These assignments should leave the host coach free to respond to unexpected requests and generally oversee the meet. The assignment sheet should also give locations for the various events.

Rooms Needed

At each meet a number of rooms are needed as indicated by the following suggestions.

- Large meeting area for students (initial announcements, announcing of events, holding area, posting of solutions and scores, etc.)
- One room or set of rooms for Events A and C; similarly for Event B.
- A room for each team (and for alternate teams if your Division allows them) for use during the team event.
- A room, preferably isolated from areas of activity, for grading. Keep exam materials here during the meet; proctors pick them up as needed. Materials should be turned over so they cannot be read while the meet is in progress.

Displaying Scores

Before the meet, prepare a scoreboard large enough to be seen at some distance. The scoreboard should list team total points accumulated during the season so far. A large marker board will do, but many Divisions use a poster which can then be awarded to the team winning that day's meet so they can display it the next day at their school. Some Divisions use other technology (overheads, monitors) for displaying results.

Copying Materials

Try to have access during the meet to a duplicating machine just in case you run short of materials.

Grading

Graders receive League-provided solutions with which to grade student tests. Graders should mark questions correct if the student answer matches the provided answer. Individual and team scores need to be compared

across the League's many Divisions with awards, scholarships, and appearances in the State Tournament dependent upon these comparisons. Therefore, it is essential that fundamental grading practices be as uniform as possible.

The following rules attempt to form a common grading foundation:

Partial credit: Unless specific instructions are given to the contrary in the official solutions, no partial credit should be given on any individual or team question.

Form of an answer: All answers will be integers.

Once the tests are graded, the scores are then entered into the Math League Online Scoring and Reporting System via the League's website.

Students may challenge an answer marked as incorrect, using the Challenge Procedure described below.

Displaying solutions

Following each event, copies of the solutions should be displayed for the students to see.

Challenges

If the official solutions contain an error, and it is discovered prior to the meet, all efforts will be made to alert the Division coordinators and coaches as to the correct answer. Coaches would then use the corrected answer to judge a response as acceptable or not.

If the official solutions contain an error, or if students believe their electronic submission has been graded incorrectly, or if the student had a unique interpretation of the question which resulted in a different answer, the students must submit a challenge to their coach no later than noon of the day ***following*** the meet.

Challenges submitted usually fall in these categories – challenges of a grading mistake by the coach (the student actually got it right but the coach mistakenly marked it wrong) or challenges caused by a unique interpretation of the question resulting in a different answer.

The challenges should be handled in this manner:

1. Challenges of mistakes made by the coach should be *fixed by the coach*.
2. Challenges as to a different answer due to a unique interpretation of the question *should be sent to the League office*.

Process for submitting a challenge to the League office:

All coaches in the Division should be made aware of the challenge. A short explanation as to the reason for the challenge, should be emailed to mathleague@augsborg.edu and to all schools in the Division by noon on the day following the meet. The student should write the explanation but may need a coach's assistance. The League will make every effort to rule on the challenge within 2 – 3 days. Schools in the Division will be notified as to the final ruling. Scores will be adjusted as necessary.

Only students who challenge a solution can be awarded points. If a student does not make a challenge, yet that student's answer is judged to be correct, that student who did not challenge will NOT be awarded the point(s) for a correct solution. Therefore, it is wiser to challenge than not.

Given this challenge process, and to ensure the most uniformity, coaches should mark answers correct only if the student answer matches the answer provided in the League answer key.

Refreshments

Some Divisions serve refreshments to participants at each meet; some do not. Such a practice does, of course, appeal greatly to the participants, and can often be funded either by school funds, the PTA, etc. The League does not provide funds for meet refreshments. Host coaches should follow practices established within their Division.

Steps For Every Meet Competing Online

- Meet Setup
 - Go to Team Admin > Team Meet Setup
 - Select the students on your scoring team and what events they will be in
 - Preselect events for all students – scoring team and alternates
- Control Competition
 - Go to Meet Op > Online Comp Control
 - Select who the recorder will be for the scoring team and optionally an alternate recorder.
 - Activate Individual Events (this year you will need to activate successive round for all your students to start each round instead of successive rounds automatically starting.)
 - Activate Team Event
- Verify Grades
 - Go to Meet Op > Grade Online Taken Events
 - May be started as soon as some students have finished their first event
 - Individual Events
 - Select each student and verify auto-scoring was correct.
 - Mark team done with event
 - Team Event
 - Select scoring team and alternates and verify auto-scoring
 - Mark team done with meet

Displaying solutions

Following each event, solutions should be displayed for the students to see. Individual schools can decide to wait until all three individual events are completed or allow students to look at solutions after each event. Team event solutions should be made available following the team event.

Training videos for online scoring system

See the three training videos (“OC Student Accounts”, “OC Control and Students” and “OC Grading”) within the Math League Online Scoring and Reporting System which describe how to run a meet and coaches’ responsibilities.

Challenges

When schools host their own meets, challenges will be conducted in the same manner as in-person meets. Please see the above section for information on challenges.

Scheduling and Postponement of Meets

All meets are scheduled on Mondays or Tuesdays. Meet dates are selected by the Board of Directors and schools are encouraged to place meets on their calendars as soon as they are published. Postponement or cancellation of a meet because of weather conditions is a decision made at the Divisional level, typically by the designated Division Coordinator, working within any guidelines the Division has established. The League Office shall be notified as soon as possible of any postponement, and the Executive Committee shall, in exercising its right to extend special State Tournament invitations to top-scoring individuals, weigh any possible effects of postponed meets.

SPECIAL CIRCUMSTANCES: The League acknowledges that special circumstances, other than weather conditions, may arise that may affect team or individual participation at a meet. Any variation from the standard meet procedures that have been set forth in this manual must obtain prior approval by the Executive Committee or its designee.

End of Season Honors and Awards

Division Coordinators, working with guidelines developed by Division Coaches, should plan a suitable awards ceremony at the conclusion of the regular season. This most often takes the form of a dinner (or a pizza party) paid for by an area industry (or by assessing each school in the Division). Division Coordinators may forward bills for their recognition event of up to \$70 per team to the League office. Most award ceremonies have been held in conjunction with Meet 5 and include all students who have participated. This format may be changed by any Division wishing to do so, but it should be remembered that goals of the League are to recognize effort and achievement in mathematics, to give increased visibility to activities available to those with interest and ability in mathematics, and to encourage students with mathematical talent to pursue further training in the discipline. The recognition event should be consistent with those goals. Media coverage is of course desirable. To assist the schools in recognizing students at the end of the year, the Board of Directors has approved the following program of awards for participants in the Minnesota State High School Mathematics League:

- The League will award plaques to the first-place team in each Division based on the cumulative ordinal scoring. In Divisions of 8 or fewer teams, a certificate suitable for framing will be awarded to the second-place team. Additionally, in Divisions of 9-12 teams, a certificate will be awarded to the third-place team; in Divisions of over 12 teams, certificates will be awarded to the third- and fourth-place teams. In addition to an engraved statement of achievement (identifying the League, the Division, and the year), each plaque will bear the names of the coach and each school participant who (1) participated as a team member or as an extra in at least three of the five regular meets, and (2) was selected at least once during the season as a member of the school's scoring team.
- Ties in total ordinal points will be broken by total season points.
- The League will award a certificate suitable for framing and a pin to the individual on each team who has over the season accumulated the most points.
- The League will provide desktop awards to students in each Division who accumulate the most points. Award items for the top three students in the Division will be selected by the League and change from year to year. All Top Division students according to the schedule below will receive pins and certificates. The

desktop awards will change from year to year in anticipation that some students may win in successive years, and the number awarded in a Division will vary with the size of the Division as follows:

Teams in Division	Desktop Award	Division Pins	Team Pin	Team Plaques
1-8	3	10	1 per school	1 + 1 certificate
9-12	4	15	1 per school	1 + 2 certificates
Over 12	5	16	1 per school	1 +3 certificates

(Unless the League Associate Director is otherwise instructed by the Division Coordinator, the awards will be sent to the Division Coordinator who should check them beforehand to see that all is in order.)

- The League will award trophies to the individuals that finish first, second, and third in individual Power Scoring (overall across the state) during the League's regular five-meet season. These awards will be presented at the State Tournament.
- The League will award trophies to the schools that finish first, second, and third in their respective Class (based on cumulative total points) during the League's regular five-meet season. These awards will be presented at the State Tournament.

Schools are also encouraged to recognize individuals who participate on their Math Team. The awards (a school letter, a pin of some kind) and the method of representation should give recognition to the student and increase school awareness of the activity. These awards (cost, decision as to who receives them) are completely the responsibility of the local school.

D. State Tournament Procedures

Overview of State Tournament Structure and Competition

The Math League season culminates with an end-of-the-year State Tournament. There are three components to the State Tournament: an Invitational Event in which top-scoring individuals from the regular season compete directly with each other, a Math Bowl competition staged as a public quick-response event between top scorers in the Invitational, and finally, a Tournament Team Contest that follows the regular season meet rules. Individuals are invited to the State Tournament on the basis of their standing in their Division or Section or overall standing in the state (see further explanation below). Teams are invited to the State Tournament on the basis of their standing in their Section or their overall standing in their Class or in the state (see further explanation below). Awards are given at the end of the meet for outstanding performances. Those who travel more than 50 miles to the tournament site may elect to be housed overnight by the League.

Tournament Invitational Event and Math Bowl

Tournament Invitational Event participants

Invitations to individuals to participate in the Tournament Invitational Event will be extended according to the following procedures.

- i. The top-scoring individual from each Division and each Section shall be invited.
- ii. From the list of top-scoring individuals in the League, ranked on a statewide basis in order of Power Scores earned during the regular season, the top 50 students shall be invited.
- iii. The Executive Committee or its designee may invite other students who because of individual circumstances may not be selected in steps 1 and 2 but who have compiled outstanding individual records.

The Tournament Invitational Event

The Invitational Event is a 30-minute test with a maximum score of 24 points. The event consists of eight quickie questions (one point each), four questions intended to be equivalent in difficulty to the higher-numbered questions that normally appear on regular season Individual Events (two points each), and two multiple-part challenge questions (four points each). This is a no-calculator event and all answers will be integers.

The top ten scorers in the Invitational Event based on Power Scores then compete in the Math Bowl.

The Math Bowl

The Math Bowl is a quick-response elimination competition that operates under the following rules and procedures.

- i. The top ten students from the Invitational Event will be selected to participate. Seasonal scores will be used to break ties.
- ii. The names of the participants will be announced at the time of the event. The participants will be asked to come to the stage and proceed to an assigned seat at a table on stage. They will be asked to print their name and school at the top of a name placard and will be given a small whiteboard on which to write their answers. A dry-erase marker and eraser will be provided.
- iii. This is a no-calculator event.
- iv. Each student will be given a written problem (one problem at a time) with enough space under the problem to do their work. A time limit is imposed on each problem and when time is called, participants hold up their answers. A point is awarded to each student displaying a correct answer.
- v. At the end of 8 problems the participant(s) with the top number of correct answers (including ties, if any) will continue and the rest of the competitors will leave the stage.
- vi. After each additional question, those who had incorrect answers will be asked to leave the stage. This will continue for as many as 7 more questions. If at the end there are still ties, the Invitational scores will be used to break the ties.
- vii. One winner is declared (if possible). The winner receives a trophy.

Tournament Team Contest

Tournament Team Participants

Invitations to teams will be extended according to the following procedure:

- i. The team with the best ordinal score from each Section is invited to the State Tournament. In addition, 2 wild card teams per Class are invited to the State Tournament. The wild card teams are the two highest raw-scoring teams in the Class that didn't win a Section.

- ii. Therefore, 10 teams from each Class receive automatic invitations. At this point, 30 teams are invited to the State Tournament. The rest of the State Tournament field is filled out by choosing the next 6 to 10 highest raw scoring teams from across the state, regardless of Class, for a full State Tournament complement of 38 teams.

Number of students per school at Tournament Team Contest

Invited teams are to bring eight team members to represent their school in the tournament. Teams wishing to bring 1 alternate may do so, but will be assessed a fee for the alternate to cover the costs of room and board. Alternates will be formed into one or more teams to compete as Alternates Team 1, Alternates Team 2, etc. in the tournament, but these teams will not be eligible for awards.

Teams requesting extra facilities (a practice room) or awards for Assistant Coaches will be assessed for extra costs incurred.

Recognition of Individual Scoring Leaders

The League will award trophies to the individuals that finish first, second, and third in individual scoring at the State Tournament. Those students who reach the Invitational Event at the tournament will receive certificates. The scores received at the Invitational together with the scores received at the tournament will determine the first, second, and third place for Tournament Scoring Leaders.

When donors provide scholarship funds, these scholarships shall be awarded to individuals ranked highest on the basis of the sum of the season total score and the tournament total score. Such awards will be deferred until after high school graduation and sent directly to the student upon League receipt of a letter indicating how the student plans to use the money to further his/her education. If said letter is not received within 3 years of graduation, the student forfeits the scholarship and the monies are returned to the scholarship fund.

Recognition of Top Scoring Teams at the Tournament

At the tournament, awards will be given for the top scoring teams in each of the Classes. In addition, an award for the team that 1) does not earn an award within any of the Classes and 2) demonstrates the most improvement from their average regular season score compared to their State Tournament score will also be presented at the State Tournament. This award is known as the Quantum Leap award and is a distinction of significant team improvement.

Tournament Weather Procedures

Owing to scheduling commitments made for hotel rooms, and for the use of the host school, our policy is to hold our tournament on the scheduled day if it is at all possible. This recognizes the fact that our tournament, once canceled, would be extremely difficult to reschedule.

If severe weather conditions seem to threaten our ability to proceed with the tournament, information shall be available via the following instruments:

- i. on our website: www.mnmathleague.org
- ii. on Twin Cities metro radio/TV channels
- iii. via email message

The Executive Committee will try to make any determination on the canceling of the tournament by 7 AM on the day of the tournament. Travel conditions typically vary across the state, as do intended modes of travel

(bus, van, private automobile). It is expected that participants in the tournament will in all cases follow the rules and directives of responsible officials of their school in deciding whether to attempt the trip to the tournament.

If participants from a school can assemble themselves locally but cannot make the trip to the tournament, they may, if arrangements are approved ahead of time by the League's Executive Committee, participate electronically. Scores obtained in this way by people taking the exams at the same time in another location shall be posted with the scores of teams at the meet, and shall qualify for awards as if they were present.

For teams and individuals unable to be present at the tournament to accept awards they have earned, the League Director shall make a good-faith effort to personally present such awards in an appropriate venue (school awards night, honors banquet, etc.). In cases where several invited schools from the same area of the state cannot get to the tournament, the League shall cooperate with said schools in setting up a suitable recognition event in their area later in the spring.

III. Other League-Sponsored Activities

A. Minnesota All-State Math Team

American Regions Mathematics League Competition (ARML)

Each spring approximately 80-100 students are invited to be participants on the Minnesota All-State Math Team, representing Minnesota at the national American Regions Mathematics League (ARML) competition. In addition, approximately 10 to 15 students from grades nine and ten are invited to be ARML “students in training.” Selection for the All-State Math Team and students in training is based on a combination of a student’s individual performance during the regular League season and performance at evaluations during Spring Practices. The top ten scorers in each of these categories are each guaranteed an All-State Math Team invitation.

A student who accepts an invitation to be a member of the Minnesota All-State Math Team has the following responsibilities:

- i. Attend 3 of four full team practices on Saturdays following the state tournament, including the New Team Member Orientation for new participants.
- ii. Participate at the ARML Competition held at the University of Iowa on the last weekend in May or the first weekend in June.
- iii. Raise \$450 to cover the cost of travel to ARML

Letters of invitation should be sent to the students via their coaches within the week following the State Tournament. A letter will also be sent to the principal of the school attended by each invitee recognizing the honored student and asking for help in raising the necessary funds.

After one or more of the practice sessions, the coaches may decide to invite one or more of the students in training to be members of the All-State Math Team. At that time, a letter will also be sent to the principal of the school attended by each invitee recognizing the honored student and asking for help in raising the necessary funds.

A total of 64 students, four teams of 15 and 4 alternates, will ultimately be selected for the All-State Math Team and travel to Iowa for the competition.

The head coach of the Minnesota All-State Math Team and three or more additional coaches will be selected by the Executive Committee. A stipend will be given to each coach.

In addition, the All State Team will represent Minnesota when possible at national contests throughout the year, including HMMT, CMIMC, PUMAC, BMT, SMT, or other relevant contests.

B. Summer Mathematics Institute (SMI)

Beginning in 2012, the League began offering a week-long summer residential program for students entering grades 7-12 called the Summer Mathematics Institute. The Institute provides an opportunity for students to experience a residential math program on a college campus where they can learn mathematical topics that are beyond the typical high school curriculum. The curriculum enables them to prepare for success in math competitions at the high school level.

IV. Other Associated Competitions

American Mathematics Competitions (AMC)

While the American Mathematics Competitions (AMC) are not an official part of our League activities, they provide an additional opportunity for our most gifted students, and we encourage League members to participate. Our meet topics align closely with those covered by the AMC 10 and AMC 12. We also encourage participation in other members of the AMC family of tests: AMC 8, the Junior High School version of AMC 10 and AMC 12, AIME (the American Invitational Mathematics Examination), USAMO (the United States of America Mathematical Olympiad), and the IMO (the International Mathematics Olympiad).

V. Resources for Coaching

A. Math League Sponsored Summer Coaches' Conference

Each summer, we invite coaches and their significant others to a conference on the Augsburg University campus. These conferences were originally supported by a grant from the Blandin Foundation to whom we had proposed that the conferences should have the following goals:

- Give specific help to coaches in some aspect of working with mathematically gifted students.
- Create an *esprit de corps* among the coaches/teachers by coming together in a congenial setting to discuss the season just past, possible changes to strengthen our League, and ways that we work with our teams.
- Make it clear, both to coaches and to their significant others, that the extra time required for League activities is recognized and appreciated by a state increasingly dependent on people able to provide leadership in mathematics and technology.

The Summer Conference is a day and a half event with work sessions during the day often followed by social activities for coaches. The second day continues with work sessions designed to help coaches with the tasks of attracting students gifted in mathematics and with all the aspects of preparing them for competition. We have brought some of the country's best-known mathematics coaches and problem solvers to the conference to present to our coaches.

Problem Database

Coaches are encouraged to use problems from previous years for student practice. Past exams are available for download in the Math League Online Scoring and Reporting System via the League website under the "Archive" tab. It should be noted that meet topics changed as of the 2024-25 season. The online database allows coaches to search for specific topics aligned to the current topic list.

How to Start a Team and other related links

Coaches who are just starting a team are encouraged to go to the League website (www.mnmathleague.org) and visit the "For Coaches" section.

VI. Governance

Having begun in 1980-81 as a group of four schools, and having grown to a group of 156 schools in 1986-87, we were, on September 10, 1987, officially incorporated under the laws of Minnesota Corporate Charter Number 18-388 with the name MINNESOTA STATE HIGH SCHOOL MATHEMATICS LEAGUE.

We include below a summary of the League Bylaws. A complete copy is available from the League Office.

MEMBERSHIP: Members are high schools in, or bordering on, the State of Minnesota. Membership is granted for an academic year and must be renewed annually. Member schools must affiliate with an existing Division of the Corporation or be assigned to a new Division by the Board of Directors. It is the intention of the board that each Division be composed of at least five teams. The Board of Directors sets dues that member schools must pay before the date of the first fall meet.

Following the State Tournament, each Division shall appoint a Division Coordinator for the following year. This Division Coordinator is a member of the Board of Directors.

DIRECTORS: The Board of Directors consists of

1. One Division Coordinator to be chosen by each Division of the Corporation, each of whom shall serve for a one (1) year term that commences with the summer conference in the year of appointment.
2. Up to five (5) members elected at large by the Board, each to serve for a three-year term
3. The League Director, elected for a five-year term, serving as an ex-officio member.
4. The All-State Math Team Coach, the Head of the Problem Writing Team, and the State Tournament Director, if not already on the board.

Directors may succeed themselves, and there are provisions for removal of any director.

MEETINGS: The annual meeting of the Board of Directors shall be held in the fall at a time agreeable to the members. Other meetings may be scheduled as needed, and the Executive Committee may call special meetings.

DECISIONS: A majority of the Directors constitute a quorum, and decisions at meetings having a quorum present shall be made by a majority of those present, unless a greater number is required by an applicable law or Robert's Rules of Order.

EXECUTIVE COMMITTEE: The President, Secretary and Treasurer of the Corporation and five (5) additional Directors shall constitute an Executive Committee of the Board. In addition, the League Director and Associate Director shall serve as non voting members of the Executive Committee. The Executive Committee exercises the power of the Board of Directors between meetings of the Board.

COMPENSATION: The League Director and such staff as the Board of Directors shall approve from time to time shall be compensated on an annual basis at rates to be set by the Board of Directors. Other members of the Board receive no compensation for services as Directors, but may be compensated for services rendered in a capacity other than that of a member of the Board.

LEAGUE MANUAL: The League Director shall prepare, or cause to be prepared, a League Manual for each academic year, which shall consist of a compilation of the rules adopted by the Board of Directors from time to time and then in effect, and which shall be subject to the approval of the Board of Directors. The League Manual shall include all rules for meets, information regarding League business and affairs, and topics to be included in the tests for the year.

AMENDMENTS: The Board of Directors may amend the Articles of Incorporation or the Bylaws at a meeting of the Board of Directors for which proper notice, stating the purpose thereof including the proposed amendment, has been given at least five (5) days in advance of the meeting. If notice required by this Article has been given, the proposed amendment or a duly modified version thereof, may be adopted at any meeting of the Board of Directors by a two-thirds (2/3) vote of the Directors present at the meeting and entitled to vote.

Officers, At-Large Board Members, and Executive Committee

Officers

Stacy Paleen, President, Suburban East Division Coordinator (elected 2022 for a 3-year term)

Jenna Innes, Treasurer (elected 2022 for a 3-year term)

Reid Froiland, Secretary, Northern Lights Division Coordinator (elected 2022 for a 3-year term)

Executive Committee Members (1-year term)

Gary Kannel, Head Technologist

Luke Olson, State Tournament Host

Zach Sheffert, Tri-Metro Division Coordinator, All-State Math Team Co-Head Coach

Colin Gardner-Springer, Head of the Problem-Writing Team

Daniel Honigs, Metro Alliance Division Coordinator

At-Large Board Members

(none)

VII. Hall of Fame

In 2005, in celebration of its 25th anniversary, the Math League instituted a Hall of Fame. The following individuals were inducted into the inaugural class:

Bill Boulger	Roger Sadlowsky	Stan Vee
Marlys Henke	Roy Schuman	Judy Cogna
Jerome Holicky	Kay Shager	Kathy Grundhoefer
Wayne Hysjulien	Bill Shimek	Stan Hill
Tom Kilkelly	Jack Sorteberg	Brant Klepel
John Kunz	Kathy Trier	Wayne Roberts

Also, to celebrate the 25th anniversary, Wayne Roberts wrote a book looking at the history of the Math League. The book is filled with anecdotes about instrumental people who helped form the League, students who have left a big impression during their tenure, and statistics from all the years the League has been in existence. That book is available by contacting the League office.

In the fall of 2005, the Board voted to select more members for the hall of fame during 2010, and every 5 years thereafter, in conjunction with the League's anniversaries. Please submit nominations to the League Office during the appropriate selection year.

The 2010 Inductees were:

Mick (Robert) Boatz	Shari Colvin	Larry Luck
Don Nitti	Tom Young	John Walther

The 2015 Inductees were:

Mike Reiners	Martha Knutson	Terry Hewitt
Karl Oleson	Ed Anderson	

The 2020 Inductees (inducted in 2021) were:

Darryl Anderson	Tracy Bibelnieks	Dan Butler
Mike Huberty	Gary Kannel	Dana Koletar
David McMayer	Curt Michener	Mary Rueter

Appendix A: Divisions and Coordinators for 2024 – 2025

Name	Division Coordinator		Name	Division Coordinator
Big 9	Jacob Johnson		Polar	David Duesler
Canterbury	Gary Kannel		Prairie	Chad Schmiesing
Central Gopher	Ben Thell		Rum River	Stephen Larson
Classic Suburban			St Louis River	Matt Solberg
Dynamic South	Aaron Perkins		St Paul City	Brian Paulson
Hiawatha	Mike Lacine		South Suburban	Chuck Croatt
Iron Range	Matt Erickson		Southwest Suburban	Kristin Johnson
Metro Alliance	Daniel Honigs		Suburban East	Stacy Paleen
Minneapolis	Jay Selvaag		Three Rivers	Allison Kaatz
Minnesota Valley	Jennifer Floren		Tri Metro	Zach Sheffert
Mississippi	Curt Michener		Twin Cities Suburban West	Ernie Johnson
Northern Lights	Reid Froiland		Wasioja	Jodi Flynn
North Suburban	Rebecca Young		West Central	Nicole Severance

Appendix B: Event Topics

2024-25 MSHSML Topics

A - Algebra	B - Geometry & Trigonometry	C - Counting, Probability & Statistics, Number Theory
1A 1. Decimals, Fractions, and Percents 2. One Variable Linear Equations & Inequalities 3. Exponent Rules 4. Square Roots and Radicals	1B 1. Angles & Angle Relationships 2. Triangle Similarity and Congruence 3. Analytic Geometry of a Straight Line 4. Trigonometry Basics	1C 1. Basic Counting 2. Statistical Measures 3. Prime Factorization & Divisibility Rules 4. GCD and LCM
2A 1. Systems of Linear Equations 2. Binomials and Quadratics 3. Absolute Value 4. The Logarithm	2B 1. Area, Perimeter, and Lengths in Triangles 2. Right Triangles 3. Analytic Geometry of Points and Lines 4. More Elementary Trigonometry	2C 1. Counting Permutations and Independent Events 2. Analyzing Data 3. Basic Probability 4. Base n Arithmetic
3A 1. The Quadratic Formula 2. Polynomials 3. Arithmetic Sequences & Series 4. Complex Number Arithmetic	3B 1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons 2. Problem Solving involving Triangles, Quadrilaterals and Polygons 3. Transformations in the Plane 4. Trigonometric Identities	3C 1. Counting Combinations 2. More Probability 3. Pascal's Triangle and the Binomial Theorem 4. Calculating Digits, especially the Last Digit
4A 1. Optimization Problems 2. Roots of Polynomial Equations 3. Geometric Sequences & Series 4. More Logarithms & Exponents	4B 1. Area, Perimeter, Angles in Circles 2. Lengths involving Circles 3. Analytic Geometry of Circles 4. Law of Sines & Law of Cosines	4C 1. More Counting & Probability 2. Expected Value 3. Remainders & Modular Arithmetic 4. Finding Integer Solutions
5A 1. Non-Linear Equations and Systems of Equations 2. Functional Equations 3. Sums of Powers of Integers 4. General Sequences & Series	5B 1. 3-Dimensional Geometry 2. Geometry Problem Solving 3. Analytic Geometry of Conic Sections 4. Geometry of Complex Numbers	5C 1. Divisor Arithmetic 2. Venn Diagrams & the Principle of Inclusion-Exclusion 3. Geometric Probability 4. More Integer Solutions

- Notes:**
- Problems may draw on topics from any previous meet.
 - Logic or puzzle type problems may appear in any event throughout the season.

Appendix C: Event Topics per Meet

Meet 1 Topics Breakdown

1A. Algebra

1A.1. Decimals, Fractions, and Percents

- Adding, subtracting, multiplying, or dividing fractions and decimals
- Reducing fractions to lowest terms
- Converting fractions to decimals
- Converting decimals (terminating or repeating) to fractions
- Percent increase & decrease
- Ratios & Proportions
- Interest problems

1A.2. One Variable Linear Equations & Inequalities

- Solving linear equations in one variable
- One variable linear inequalities
- Word problems leading to linear equations or inequalities
- Rate problems (distance = rate \times time)
- Unit Conversion

1A.3. Exponent Rules

- Positive integer exponents
- $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-n} = \frac{1}{a^n}$, $a^0 = 1$

1A.4. Square Roots & Radicals

- Square roots
- Cube roots & higher roots
- Simplifying square roots & radicals of integers
- Adding, subtracting, multiplying, and dividing radicals
- Rationalizing simple radical denominators
- Radicals as exponents: $\sqrt[n]{a} = a^{\frac{1}{n}}$

1B. Geometry & Trigonometry

1B.1. Angles & Angle Relationships

- Angle sums in triangles and polygons
- Parallel lines and angle relationships

1B.2. Triangle Similarity and Congruence

- Conditions for congruence: SSS, SAS, ASA, AAS
- Conditions for similarity: AA, SAS, SSS
- Problem solving in triangles using Similarity and Congruence

1B.3. Analytic Geometry of a Straight Line

- Slope
- Slope-intercept form of a straight line
- Point-slope form of a straight line
- Midpoint of a segment

1B.4. Trigonometry Basics

- Degrees, radians, and converting between the two
- Right Triangle trig definitions
- Unit Circle trig definitions

1C. Counting, Prob. & Stats, Num. Theory

1C.1. Basic Counting

- Counting lists
- Counting by cases
- Complementary counting

1C.2. Statistical Measures

- Average/mean
- Median
- Mode

1C.3. Prime Factorization & Divisibility Rules

- Testing for divisibility by 2 through 12 (except 7)
- Prime factorization (using divisibility rules and testing primes up to square root)

1C.4. GCD and LCM

- Calculating the Greatest Common Divisor by factoring
- Calculating the Least Common Multiple by factoring
- Calculating Greatest Common Divisor using the Euclidean Algorithm
- $\gcd(a, b) \times \text{lcm}(a, b) = ab$

Meet 2 Topics Breakdown

2A. Algebra

2A.1. Systems of Linear Equations

- Two (or occasionally more) variable linear systems of equations
- Word problems leading to systems of equations

2A.2. Binomials and Quadratics

- Multiplying binomials
- Rationalizing denominators using conjugates
- Difference of squares factorization
- Factoring a quadratic as a product of binomials
- Solving quadratic equations by factoring

2A.3. Absolute Value

- Solving absolute value equations and inequalities in one variable
- Representation on the Number Line

2A.4. The Logarithm

- Definition of logarithm
- Relationship to exponents
- $\log_n(ab) = \log_n a + \log_n b$
- $\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$
- $\log_n(a^k) = k \log_n a$

2B. Geometry & Trigonometry

2B.1. Area, Perimeter, and Lengths in triangles

- Base-height area formula
- Medians, angle bisectors, altitudes
- Triangle Inequality
- Heron's Formula for triangle area
- Triangle theorems: Angle Bisector, Stewart, Ceva, Menelaus

2B.2. Right Triangles

- The Pythagorean theorem
- 30-60-90 and 45-45-90 triangles
- Common Pythagorean Triples

2B.3. Analytic Geometry of Points and Lines

- Distance between points
- Finding intersection points of lines
- Systems of inequalities used to define a region in the plane
- Areas of polygons on a grid
- Distance from a point to a line

2B.4. More Elementary Trigonometry

- Trig functions of common angles
- Basic identities: $\sin^2 \theta + \cos^2 \theta = 1$,
 $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$,
 $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$, etc.

- Graphs of trig functions (sin, cos, tan)
- Inverse trigonometric functions
- Triangle area using trigonometry:
 $A = \frac{1}{2}ab \sin C$
- Solving trigonometric equations

2C. Counting, Probability & Statistics, Number Theory

2C.1. Counting Permutations and Independent Events

- The multiplication principle for counting
- Counting Permutations: ${}_n P_r$
- Factorials
- Knowing when to add and when to multiply

2C.2. Analyzing Data

- Analyzing data in tables
- Analyzing data in charts
- Analyzing data in graphs

2C.3. Basic Probability

- Definition of probability
- Calculating probabilities using basic counting & permutations

2C.4. Base n Arithmetic

- Base n numbers and base n arithmetic
- Converting base n numbers to and from base 10
- Converting base n numbers to and from other bases

Meet 3 Topics Breakdown

3A. Algebra

3A.1. The Quadratic Formula

- Solving quadratics by completing the square
- Using the Quadratic Formula to solve quadratic equations
- The discriminant and character of roots
- Quadratic Inequalities

3A.2. Polynomials

- Multiplying polynomials
- Finding integer or rational roots of polynomials (the Rational Root Theorem)
- Factoring polynomials based on a known root; The Remainder Theorem
- Sum and difference of cubes factorization
- Sum and difference of odd powers factorization
- Solving polynomial equations
- Simplifying rational expressions (including Polynomial Division)
- Solving rational equations

3A.3. Arithmetic Sequences and Series

- Arithmetic sequence definition
- Finding the common difference
- Finding the n th term
- Arithmetic series definition
- Calculating the sum of an arithmetic series
- Arithmetic sequence & series problem solving

3A.4. Complex Number Arithmetic

- Adding and Subtracting complex numbers
- Multiplying complex numbers
- The Complex Conjugate
- Dividing complex numbers

3B. Geometry & Trigonometry

3B.1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons

- Squares, rectangles, parallelograms, the rhombus, trapezoids, & other quadrilaterals
- Computing area, perimeter, & lengths
- Polygons (regular and otherwise)

3B.2. Problem Solving Involving Triangles, Quadrilaterals and Polygons

- Using all geometry topics covered to date
- Specifically covers anything from 1B.1, 1B.2, 2B.1, 2B.2, and 3B.1

3B.3. Transformations in the Plane

- Scaling
- Reflections, typically across horizontal or vertical lines
- Rotations, typically by multiples of 90 degrees
- Relationship between line slope and the tan function

3B.4. Trigonometric Identities

- Angle sum and difference formulas
- Double angle identities
- Half angle identities
- Solving trigonometric equations

3C. Counting, Probability & Statistics, Number Theory

3C.1. Counting Combinations

- Correcting for overcounting
- Counting combinations: $\binom{n}{r}$

3C.2. More Probability

- Calculating probabilities using combinations & other counting techniques
- Conditional probability

3C.3. Pascal's Triangle and the Binomial Theorem

- Pascal's Triangle
- The Binomial Theorem: coefficients of $(x + y)^n$

3C.4. Calculating Digits, especially the Last Digit

- Finding the last digit of a sum, product, or power of integers
- Counting trailing zeros of products of integers

Meet 4 Topics Breakdown

4A. Algebra

4A.1. Optimization Problems

- Minimum or maximum value of quadratic expressions (e.g. by completing the square)
- The Arithmetic-Geometric mean inequality

4A.2. Roots of Polynomial Equations

- Vieta's formula for the sum of roots of a polynomial
- Vieta's formula for the product of roots of a polynomial
- Vieta's formula for relationships between roots and other coefficients of a polynomial

4A.3. Geometric Sequences and Series

- Geometric sequence definition
- Finding the common ratio
- Finding the n th term
- Geometric series definition
- Calculating the sum of a geometric series
- Geometric sequence & series problem solving

4A.4. More Logarithms & Exponents

- The change of base formula: $\log_a b = \frac{\log_c b}{\log_c a}$
- Solving exponential equations
- Solving logarithmic equations

4B. Geometry & Trigonometry

4B.1. Area, Perimeter, and Angles in Circles

- Sectors
- Circular segments
- Central & inscribed angles
- The Inscribed Angle Theorem

4B.2. Lengths Involving Circles

- Internal and External tangent lines
- Power of a Point
- Cyclic quadrilaterals
- Angle relationships in cyclic quadrilaterals (e.g. opposite angles)
- Problem solving in cyclic quadrilaterals (e.g. Ptolemy's Theorem, Brahmagupta's Formula)

4B.3. Analytic Geometry of Circles

- Circle equations
- Intersections between circles

4B.4. Law of Sines & Law of Cosines

- The Law of Sines
- The Law of Cosines
- Finding lengths and angles in triangles and other geometric figures using these laws

4C. Counting, Probability & Statistics, Number Theory

4C.1. More Counting & Probability

- Counting integer solutions to $x_1 + x_2 + \dots + x_n = k$ ("sticks and stones"), including positive and non-negative cases
- Counting more than two groups (multinomials)
- Counting paths in a grid
- Counting & probability problems using all topics to date

4C.2. Expected Value

- Definition based on individual probabilities
- Expected value problem solving

4C.3. Remainders & Modular Arithmetic

- Calculating remainders
- Modular arithmetic notation
- Remainders of sums, products, and powers of integers

4C.4. Finding Integer Solutions

- Finding integer solutions by factoring & casework
- Finding integer solutions by completing the rectangle ("Simon's Favorite Factoring Trick")

Meet 5 Topics Breakdown

5A. Algebra

5A.1. Non-Linear Equations and Systems of Equations

- Solving radical equations
- Identifying extraneous roots
- Solving more complex equations or systems of equations (combining any techniques used to date)

5A.2. Functional Equations

- Solving for a function based on given properties of the function
- Finding function value for a specific input given properties of the function
- Domain and Range; Function Composition; Inverse Functions
- Functional Operations (Invented operators)

5A.3. Sums of Powers of Integers

- Formula for $1 + 2 + \dots + n$
- Formula for $1^2 + 2^2 + \dots + n^2$
- Formula for $1^3 + 2^3 + \dots + n^3$
- Finding related sums using these formulae

5A.4. General Sequences & Series

- The Fibonacci sequence
- Recursively defined sequences
- Telescoping sums
- General sequence & series problem solving

5B. Geometry & Trigonometry

5B.1. 3-Dimensional Geometry

- Triangular and polygonal prisms & pyramids, cylinders, cones, spheres
- Calculating volume and surface area

5B.2. Geometry Problem Solving

- Problem solving using all Geometry topics
- Could include any topics from subtopics 1 and 2 of previous B events
- Specifically topics 1B.1, 1B.2, 2B.1, 2B.2, 3B.1, 3B.2, 4B.1, 4B.2

5B.3. Analytic Geometry of Conic Sections

- Axis of symmetry
- Focus-directrix definition of a parabola
- Parabola equations
- Focus-focus definition of an ellipse
- Ellipse equations
- Focus-focus definition of a hyperbola
- Hyperbola equations

5B.4. Geometry of Complex Numbers

- The Complex Plane
- Modulus of Complex Numbers
- Distance in the Complex Plane
- Polar representation of a complex number, relationship to trigonometry
- De Moivre's Theorem

5C. Counting, Probability & Statistics, Number Theory

5C.1. Divisor Arithmetic

- Counting divisors of a number
- Sum of divisors of a number

5C.2. Venn Diagrams & the Principle of Inclusion-Exclusion

- Counting with Venn diagrams (2 or 3 sets)
- Counting using the principle of inclusion-exclusion (any number of sets)

5C.3. Geometric Probability

- Calculating probability as a fractional area

5C.4. More Integer Solutions

- More general diophantine equations
- Solving modular equations and systems

Appendix D: Formula Sheet

ANGLE BISECTOR THEOREM:

$$\frac{b}{c} = \frac{m}{n}$$

CEVA'S THEOREM:

$$\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$$

MENELAUS' THEOREM:

$$\left| \frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} \right| = 1$$

STEWART'S THEOREM:

$$b^2m + c^2n = a(d^2 + mn)$$

MORE TRIANGLE RESULTS:

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \left(s = \frac{a+b+c}{2} \right)$$

LOGARITHMS:

$y = \log_b x$ means $b^y = x$

$\log_n(ab) = \log_n a + \log_n b$

$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$

$\log_n(a^k) = k \log_n a$

$\log_a b = \frac{\log_c b}{\log_c a}$

SOME 3D GEOMETRY:

- Pyramids and cones with base area B and height h have volume $V = \frac{1}{3}Bh$.
- Volume of a sphere is $V = \frac{4}{3}\pi r^3$
- Surface area of a sphere is $SA = 4\pi r^2$

QUADRATIC FORMULA:

Roots of $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

COUNTING OBJECTS:

Given n distinguishable objects:

- The # of ways to line them up in a row is $n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$
- The # of ways to pick a set of r where order matters is $nPr = \frac{n!}{(n-r)!}$
- The # of ways to pick a set of r where order doesn't matter is $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

AM-GM INEQUALITY:

If $a_i \geq 0$, then $\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$

BINOMIAL THEOREM:

$$(x + y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

SUM AND DIFFERENCE OF POWERS:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

COMPLEX NUMBERS:

Define $i = \sqrt{-1}$, so $i^2 = -1$. Let $z = a + bi$. Then $\bar{z} = a - bi$ is the complex conjugate of z , and $|z| = \sqrt{a^2 + b^2}$ is the modulus of z .

De Moivre's Theorem:
 If $z = r(\cos \theta + i \sin \theta)$, then $z^n = r^n(\cos(n\theta) + i \sin(n\theta))$

SUMS OF POWERS OF INTEGERS:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

ANALYTIC GEOMETRY OF LINES:

- The line joining (x_1, y_1) to (x_2, y_2) has slope $\frac{y_2 - y_1}{x_2 - x_1}$
- The equation of line through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$
- The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

ANALYTIC GEOMETRY OF CONIC SECTIONS:

- A **parabola** is the set of points equidistant from a point (the focus) and a line (the directrix)
- A parabola with vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$ has equation $y = 4px^2$.
- An **ellipse** is the set of points whose distances to two fixed points (its foci) add to a constant.
- An ellipse centered at $(0, 0)$ with vertices $(\pm a, 0)$ and covertices $(0, \pm b)$ has area πab and equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A **hyperbola** is the set of points whose distances to two points (its foci) have a constant difference.
- A hyperbola centered at $(0, 0)$ with vertices $(\pm a, 0)$ and asymptotes $y = \pm \frac{b}{a}x$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.

CYCLIC QUADRILATERALS:

ABCD is Cyclic

$\Leftrightarrow \angle ADB = \angle ACB$

$\Leftrightarrow \angle ABC + \angle ADC = 180^\circ$

Inscribed Angle Theorem:
 $\angle AOB = 2\angle ADB$

Ptolemy's Theorem: $ac + bd = AC \cdot BD$

Brahmagupta's Formula:

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \quad \left(s = \frac{a+b+c+d}{2} \right)$$

TRIGONOMETRY DEFINITIONS:

Right triangle (SOH-CAH-TOA) & Unit Circle definitions:

$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$

Graphs: (π radians = 180°)

SOME TRIGONOMETRIC IDENTITIES:

$\tan \theta = \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta = 1$

$\cot \theta = \frac{1}{\tan \theta}, \sec \theta = \frac{1}{\cos \theta}, \csc \theta = \frac{1}{\sin \theta}$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin(2A) = 2 \sin A \cos A$

$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$

$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$

$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$

$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$

2024-25 Formula Sheet