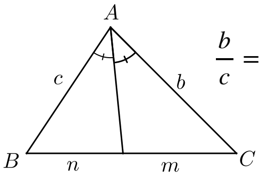
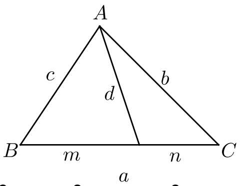
2024-25 Formula Sheet

ANGLE BISECTOR THEOREM:



$$\frac{b}{c} = \frac{m}{n}$$

STEWART'S THEOREM:



$$b^2 m + c^2 n = a(d^2 + mn)$$

LOGARITHMS:

$y = \log_b x$ means $b^y = x$

$$\log_n(ab) = \log_n a + \log_n b$$

$$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$$

$$\log_n(a^k) = k \log_n a$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

QUADRATIC FORMULA:

Roots of $ax^2 + bx + c = 0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

AM-GM INEQUALITY:

If $a_i \geq 0$, then

$$\frac{a_1 + a_2 + \ldots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \ldots a_n}$$

SUM AND DIFFERENCE OF POWERS:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

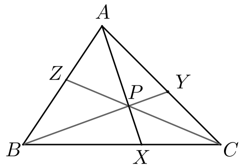
SUMS OF POWERS OF INTEGERS:

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

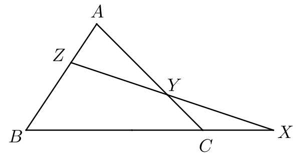
$$1^3 + 2^3 + 3^3 + \ldots + n^3 = \frac{n^2(n+1)^2}{4}$$

CEVA'S THEOREM:



$$\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$$

MENELAUS' THEOREM:



$$\left|\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA}\right| = 1$$

MORE TRIANGLE RESULTS:

Law of Sines:

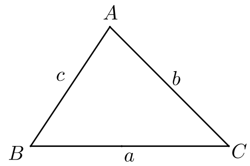
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Heron's Formula:

Area = $\sqrt{s(s-a)(s-b)(s-c)}$, $\left(s = \dfrac{a+b+c}{2}\right)$



SOME 3D GEOMETRY:

- Pyramids and cones with base area B and height h have volume $V = \frac{1}{3}Bh$.
- Volume of a sphere is $V = \frac{4}{3}\pi r^3$
- Surface area of a sphere is $SA = 4\pi r^2$

CYCLIC QUADRILATERALS:

ABCD is Cyclic

$\iff \angle ADB = \angle ACB$

$\iff \angle ABC + \angle ADC = 180°$

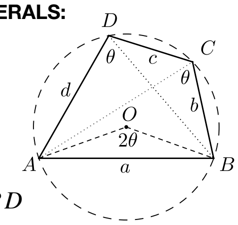
Inscribed Angle Theorem:

$\angle AOB = 2\angle ADB$

Ptolemy's Theorem: $ac + bd = AC \cdot BD$

Brahmagupta's Formula:

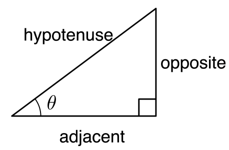
Area = $\sqrt{(s-a)(s-b)(s-c)(s-d)}$, $\left(s = \dfrac{a+b+c+d}{2}\right)$

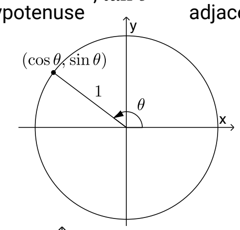


TRIGONOMETRY DEFINITIONS:

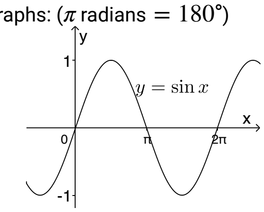
Right triangle (SOH-CAH-TOA) & Unit Circle definitions:

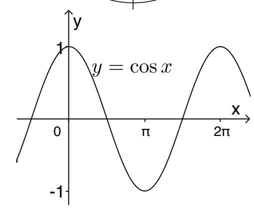
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}, \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$





Graphs: (π radians $= 180°$)





COUNTING OBJECTS:

Given n distinguishable objects:

- The # of ways to line them up in a row is $n! = n \cdot (n-1) \cdot \ldots \cdot 3 \cdot 2 \cdot 1$
- The # of ways to pick a set of r where order matters is ${}_nP_r = \dfrac{n!}{(n-r)!}$
- The # of ways to pick a set of r where order doesn't matter is $\dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

BINOMIAL THEOREM:

$$(x+y)^n = \binom{n}{n}x^n + \binom{n}{n-1}x^{n-1}y + \ldots + \binom{n}{1}xy^{n-1} + \binom{n}{0}y^n$$

COMPLEX NUMBERS:

Define $i = \sqrt{-1}$, so $i^2 = -1$. Let $z = a + bi$.

Then $\bar{z} = a - bi$ is the complex conjugate of z,

and $|z| = \sqrt{a^2 + b^2}$ is the modulus of z.

De Moivre's Theorem:

If $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$

SOME TRIGONOMETRIC IDENTITIES:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}, \sin^2\theta + \cos^2\theta = 1$$

$$\cot\theta = \frac{1}{\tan\theta}, \sec\theta = \frac{1}{\cos\theta}, \csc\theta = \frac{1}{\sin\theta}$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos a\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\sin(2A) = 2\sin A\cos A$$

$$\cos(2A) = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan(2A) = \frac{2\tan A}{1 - \tan^2 A}$$

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A} = \frac{1-\cos A}{\sin A}$$

ANALYTIC GEOMETRY OF LINES:

- The line joining (x_1, y_1) to (x_2, y_2) has slope $\dfrac{y_2 - y_1}{x_2 - x_1}$
- The equation of line through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$
- The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

ANALYTIC GEOMETRY OF CONIC SECTIONS:

- A p**arabola** is the set of points equidistant from a point (the focus) and a line (the directrix)
- A parabola with vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$ has equation $y = 4px^2$.
- An **ellipse** is the set of points whose distances to two fixed points (its foci) add to a constant.
- An ellipse centered at $(0, 0)$ with vertices $(\pm a, 0)$ and covertices $(0, \pm b)$ has area πab and equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A **hyperbola** is the set of points whose distances to two points (its foci) have a constant difference.
- A hyperbola centered at $(0, 0)$ with vertices $(\pm a, 0)$ and asymptotes $y = \pm\dfrac{b}{a}x$ has equation $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.