Minnesota State High School Mathematics League

2024-25 Sample Meet 3.2, Individual Event A

15 minutes

Score	Check

1. _____ Each of the numbers from 1 to 6 is entered into the "magic triangle" puzzle shown in Figure 1 so that the three numbers along each edge add to the same value, its "magic sum". Three numbers have already been entered. What is the magic sum for this triangle?

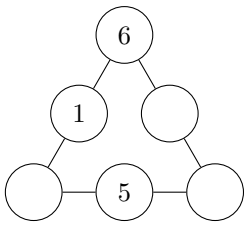

Figure 1

2. _____ Find the sum of the following arithmetic series:

$$(-20) + (-19) + (-18) + \cdots + 21 + 22 + 23$$

3. $n =$ _____ Find the value of n for which there is *exactly one* number satisfying:

$$(x - 7)(x - 11) = n$$

4. $a + b =$ _____ There are three real numbers r for which $(r - 3)$ and $\left(r - \dfrac{6}{r}\right)$ are reciprocals; that is, their product is 1. One such number is 2, but the largest of the three can be written as $\dfrac{a + \sqrt{b}}{2}$, where a and b and are positive integers. Find $a + b$.

5. $m + n =$ _____ An *Eisenstein Integer* is a complex number of the form $a + b\omega$, where a and b are integers and ω is a non-real cube root of unity (i.e. $\omega^3 = 1$ but $\omega \neq 1$). If $(2 + 3\omega)(1 - 5\omega) = m + n\omega$, determine the value of $m + n$.

Name: _____ Team: _____

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2024-25 Sample Meet 3.2, Individual Event B

15 minutes

Score	Check

1. _____ inches^2 = Kaitlyn has a 5×8 inch index card. If she shortens the length of one side of this card by 2 inches, the card would be 30 square inches. What would the area of the card be in square inches if instead she shortens the length of the perpendicular side by 2 inches?

2. _____ In square $ABCD$ with side length 10, X lies on side $\overline{AB}$ such that $AX = 5$, as seen in Figure 2. Find the area of trapezoid $XBCD$.

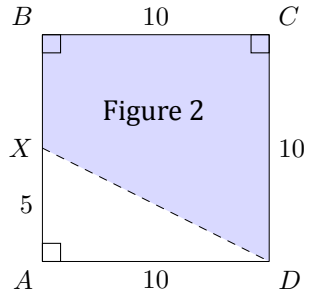

Figure 2

3. $p + q =$ _____ The value of $\sqrt{\dfrac{1 + \cos 120^\circ}{2}} = \dfrac{p}{q}$, where p and q are relatively prime positive integers. Determine the value of $p + q$.

4. $\angle P =$ _____ In Figure 4, $ABCDEFGHI$ is a regular nonagon (9-sided polygon). Secants $\overline{IB}$ and $\overline{FC}$ intersect at P. What is the measure of acute $\angle P$, in degrees?

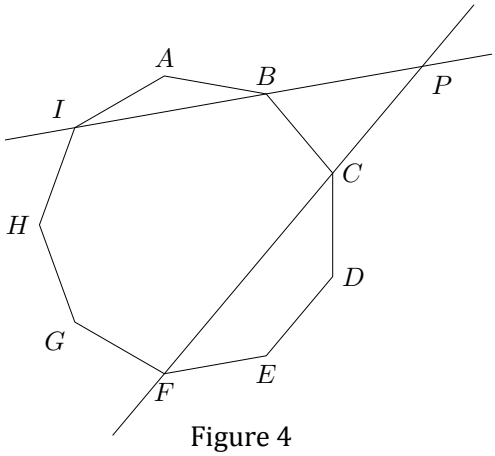

Figure 4

5. $m + b =$ _____ Line ℓ_1 has the equation $2x + 3y = 24$ and line ℓ_2 has the equation $3x + 2y = 6$. Line ℓ_3 has the equation $y = mx + b$, where $m > 0$. If ℓ_1 is the reflection of ℓ_2 with respect to ℓ_3, determine the value of $m + b$.

Name: _____ Team: _____

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2024-25 Sample Meet 3.2, Individual Event C

15 minutes

Score	Check

1. $b =$ _____ $(2x+1)^3$ can be expanded as ax^3+bx^2+cx+d where a, b, c, and d are integers. Determine the value of b.

2. _____ The integer 31415 includes 314 as a block of digits, but 30104 doesn't. How many five digit positive integers (not starting with the digit 0) include the block of digits 314?

3. $m+n =$ _____ A fair coin is tossed three times. The probability of getting exactly two heads is $\frac{m}{n}$ where m and n are relatively prime positive integers. Determine $m+n$.

4. $ab =$ _____ Given positive integers a and b, the units digit of b is 8, but Ralph thought it was 6 and got 4740 for the product of a and b. Natalie thought the units digit of b was 3 and got 4695 for the product. Determine the correct value of the product ab.

5. _____ edges A three-dimensional cube has 12 edges. How many edges does a 7-dimensional cube have?

   Hint: Consider an n-dimensional cube with vertices at $(x_1, x_2, x_3, \ldots, x_n)$ in n-dimensional space, where each x_k is either 0 or 1. Then each edge connects vertices which differ in exactly one coordinate.

Name: _____ Team: _____

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2024-25 Sample Meet 3.2, Team Event

30 minutes

Score	Check

1. _____ cm On the screen of my graphing calculator point $A\left(1, \frac{1}{2}\right)$ is 0.2 cm from the origin. In centimeters, how far from the origin would point $B(5, 10)$ be on this calculator?

2. _____ Simplify:

$$\sqrt{1} - \sqrt{1+3} + \sqrt{1+3+5} - \sqrt{1+3+5+7} + \cdots + \sqrt{1+3+5+\cdots+17} - \sqrt{1+3+5+\cdots+17+19}.$$

3. _____ Find the greatest solution to $(x+2)^5 + (x-2)^5 = 122x^3$.

4. $n =$ _____ Let $f(x) = \left(\frac{n}{4}\right)x^2 + (n+1)x + (n-2)$. Find the least integer n greater than 2021 such that $f(x)$ has rational roots.

5. _____ Five distinct numbers are selected randomly and without replacement from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. The probability that the median of these five numbers is 5 can be written as $\frac{m}{n}$ where m and n are relatively prime positive integers. Determine $m+n$.

6. _____ How many solutions does the equation

$$\sin^2\theta + \sin^2(2\theta) + \sin^2(3\theta) = 2$$

have, where $0 \leq \theta < 2022\pi$?

Team: _____

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2024-25 Sample Meet 3.2, Answers

Event A:

1. **11**
   (21-22 5A1, 84% correct)
2. **66**
   (22-23 4C1, 77% correct)
3. **-4**
   (23-24 1D2, 32% correct)
4. **38**
   (23-24 1D3, 14% correct)
5. **25**
   (21-22 3C4, 4.5% correct)

Event B:

1. **24**
   (21-22 5D1, 90% correct)
2. **75**
   (23-24 3B1, 88% correct)
3. **0**
   (20-21 2C1, 62% correct)
4. **40**
   (20-21 5B1, 32% correct)
5. **19**
   (20-21 2D4, 3.8% correct)

Event C:

1. **12**
   (21-22 4C2, 79% correct)
2. **280**
   (23-24 SC2, 80% correct)
3. **11**
   (21-22 5C1, 54% correct)
4. **4770**
   (15-16 5A3)
5. **448**
   (22-23 ST4, 68% correct)

Team Event:

1. **2**
   (20-21 SB1, 71% correct)
2. **-5**
   (20-21 5D1, 64% correct)
3. **4**
   (22-23 SI10, 68% correct)
4. **2070**
   (20-21 ST6, 57% correct)
5. **25**
   (21-22 SC4, 15% correct)
6. **10110**
   (21-22 SI13, 4.3% correct)