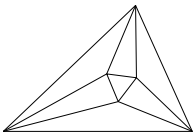1. _____ Determine the sum of all solutions to the equation $x^2 - 7x + 10 = 0$.

2. _____ Determine the value of $\log_4\left(\log_8\left(2^{768}\right)\right)$.

3. _____ Let a and b be the solutions to the equation $x^2 + x - 1 = 0$. Find the value of $(a+3)(b+3)$.

4. $r =$ _____ The 1st, 5th and 17th terms of an increasing arithmetic sequence are consecutive terms of a geometric sequence. Determine the common ratio r of this geometric sequence.

5. _____ Determine the *least* possible value of $x \cdot y$ among all real solutions (x, y, z) to the system:

$$x + y + z = 16$$
$$x^2 + y^2 + z^2 = 100$$

Name: _____   Team: _____

1. radius = _____

A larger circle with radius 6 is divided into four regions A, B, C, and D, each of equal area, as shown in Figure 1. What is the radius of the smaller circle D?

Figure 1

2. $PB =$ _____

A circle with radius 9 has tangent $\overline{PA}$ with length 40 as shown in Figure 2. $\overline{PB}$ is also tangent to the circle, at B. Find the length of $\overline{PB}$.

Figure 2

3. _____

Eight identical square sticky notes, numbered 1 through 8, are arranged to form the pattern shown in Figure 3. Notes are placed in reverse order (starting with #8, then #7, etc.) - possibly overlapping but never underneath any previously placed notes. Only note #1, indicated, is completely visible. What number goes on the partially visible sticky note indicated with a question mark?

Figure 3

4. radius = _____

Determine the radius of the circle passing through $A(-6, 6)$ and $B(18, 6)$ which is tangent to the x-axis.

5. $a =$ _____

In isosceles right triangle ABC, with $BA = BC$, point D is on $\overline{AB}$ and E is on $\overline{AC}$, such that $\frac{AD}{DB} = \frac{CE}{AE} = \frac{1}{2}$. The value of $\cos(\angle CED)$ can be written as $\frac{a\sqrt{10}}{10}$ for some positive integer a. Find a

Name: _____   Team: _____

1. _____ At a carnival, 5 tickets can be redeemed for a prize. When Matthew redeems his tickets he ends up with 2 tickets left over. When Angelica redeems her tickets she ends up with 4 tickets left over. If, instead, Matthew and Angelica had pooled their tickets to get as many prizes as possible, how many tickets would they have had left over?

2. _____
[Source: MATHCOUNTS] A bag contains 25 tickets, each colored either red or yellow. Red tickets are worth $0.50 and yellow tickets are worth $5.00. If the expected value of a ticket drawn at random from this bag is $3.20, how many tickets are red?

3. _____ Find the number of ordered triples (a, b, c) of positive integers for which

$$a + b + c = 7.$$
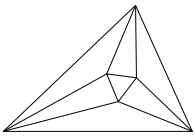
Three such triples are $(1, 1, 5)$, $(4, 1, 2)$, *and* $(5, 1, 1)$.

4. _____ An integer, increased by the value of its cube, is 592,788. What is the original integer?

5. _____ Certain real numbers r can be expressed as $r = \dfrac{n^2}{n - 4}$ for two different positive integers n. Determine the sum of all possible values of r.

Name: _____   Team: _____

1. $k =$ _____ If $161 \equiv k \bmod 13$, where $0 \le k < 13$ is an integer, find k.

2. $\angle AEB =$ _____ What is the degree measure of $\angle AEB$ based on the information provided in Figure 2?

Figure 2
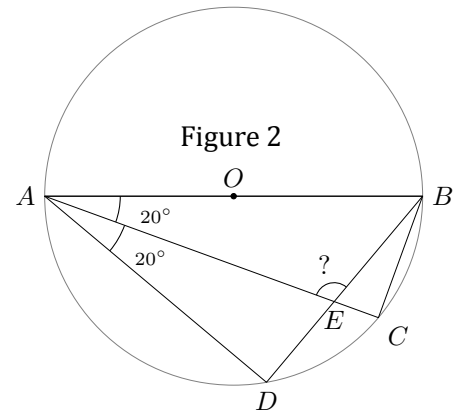
3. _____ cents Alex decides to bake jumbo cookies to raise money for his school's Math Team. The ingredients for each cookie cost 20 cents. He'd sell 70 cookies if he charged 60 cents per cookie, but only 35 cookies if he charged $\$2.00$ per cookie. Given that the number of cookies sold is a linear function of the price charged, how much (in cents) should Alex charge for each cookie to maximize *profit* for his school's Math Team?

4. _____ Square $ABCD$ is inscribed in a circle. Point E is on the arc $\overset{\frown}{AD}$ such that $ABCDE$ forms a pentagon. If $AB = 7$, determine the value of $(AE)^2 + (BE)^2 + (CE)^2 + (DE)^2$.

5. $a + b =$ _____ Suppose $2 + 3i$ is a root of the quartic equation $x^4 + ax^2 + b = 0$, where a and b are real numbers. Find $a + b$.

6. _____ Find a four-digit whole number (which can't start with 0) whose square ends with the same last four digits in the same order. For instance 25 is a two-digit equivalent, since $25^2 = 625$, which ends in 25.

Team: _____
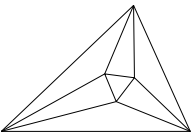
Minnesota State High School Mathematics League
2024-25 Sample Meet 4.1, Answers

Event A:

1. **7**
(21-22 1D1, 86% correct)

2. **4**
(21-22 SI5, 94% correct)

3. **5**
(23-24 SD2, 67% correct)

4. **3**
(21-22 4C3, 42% correct)

5. **14**
(21-22 5A4, 3.3% correct)

Event B:

1. **3**
(21-22 4B1, 85% correct)

2. **40**
(22-23 4B1, 66% correct)

3. **5**
(21-22 5T1, 87% correct)

4. **15**
(21-22 2D3, 14% correct)

5. **-3**
(20-21 3T4, 27% correct)

Event C:

1. **1**
(22-23 SI1, 96% correct)

2. **10**
(18 MATHCOUNTS School Sprint 23)

3. **15**
(23-24 5C2, 54% correct)

4. **84**
(11-12 1A4)

5. **43**
(21-22 1D4, 6.8% correct)

Team Event:

1. **5**
(11-12 1A2)

2. **110**
(21-22 SI6, 83% correct)

3. **180**
(22-23 4D3, 9.1% correct)

4. **196**
(20-21 4T2, 33% correct)

5. **179**
(23-24 1T5, 24% correct)

6. **9376**
(22-23 5T5, 11% correct)