Minnesota State High School Mathematics League

Score	Check

1. $A = $ _____

The equation $x^2 - 9x + A = 0$ has one root which is twice the other root. Determine the value of A.

2. $c = $ _____

If $32, a, b, c, 162$ form a geometric progression, what is the value of c?

3. _____

Determine the *sum* of the values of x such that

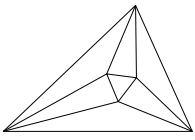$$f(x) = \frac{4}{(x-6)^2} + \frac{(x-6)^2}{4}$$

takes on a minimum value.

4. $m + n = $ _____

The solution to $5^x + 5^{x+1} = 6\sqrt{5}$ can be written as $x = \dfrac{m}{n}$, where m and n are relatively prime positive integers. Determine the value of $m + n$.

5. _____

Suppose $N = \left\lfloor \dfrac{10^{2024}}{10^{88} - 7} \right\rfloor$. Find the remainder when N is divided by 100.

(Recall that $\lfloor x \rfloor$ is defined to be the greatest integer which is less than or equal to x.)

Name: _____   Team: _____

1. _____ A circle centered at $(9, -12)$ passes through the origin $(0,0)$. What is its radius?

2. $k =$ _____ The area of the $120°$ sector of a circle with radius of 3, shown in Figure 2, is $k\pi$. Find k.

Area $= k\pi$

$120°$

$r = 3$
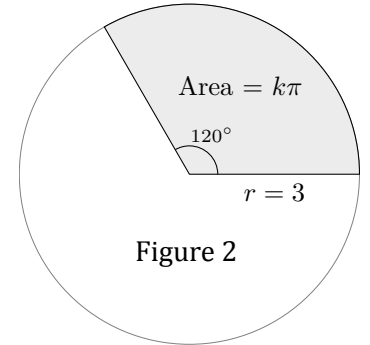
Figure 2

3. $BC =$ _____ Figure 3 shows triangle ABC with $\sin A = \dfrac{3}{4}$, $\sin B = \dfrac{1}{4}$, and $AC = 8$. Determine the length of side $\overline{BC}$.
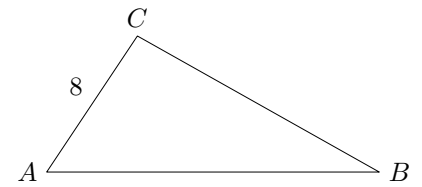
Figure 3

4. _____ Equilateral triangle ABC with side length 30 is inscribed in circle Ω. D is a point on minor arc $\overset{\frown}{AC}$ of Ω so that $BD = 33$. Determine the *perimeter* of quadrilateral $ABCD$.

5. $CP =$ _____ The *Fermat Point* of right triangle ABC is the interior point P such that $\angle APB = \angle BPC = \angle CPA = 120°$. If $\angle ABC = 90°$, $AP = 12$, and $BP = 8$, determine the length of $\overline{CP}$.

Figure 5
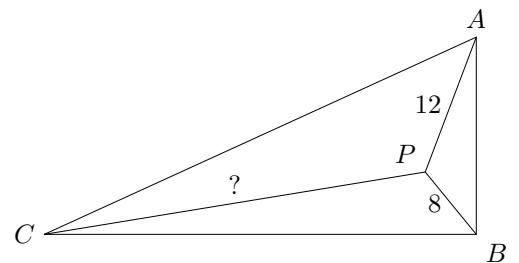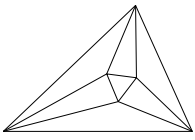
Score	Check

1. _____

Each of the numbers 1, 2, 3, 4, 5, and 6 is placed exactly once in a circle of Figure 1 so that pairs of numbers joined by a line always add to a prime number. The numbers 5 and 6 have been placed in the top two circles, and you'll note that $5+6=11$, a prime number. What number must be placed in the circle opposite 6 (indicated with a question mark)?

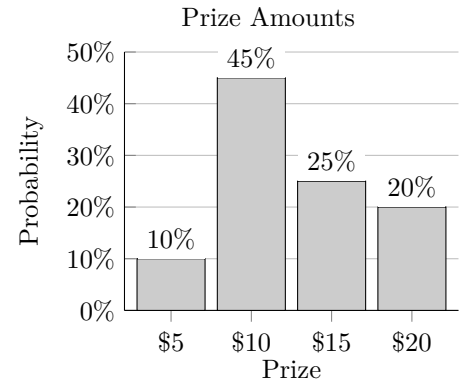Figure 1
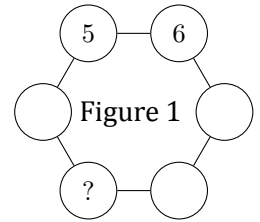
2. $X =$ _____

The six-digit number $\underline{2}\,\underline{1}\,\underline{7}\,\underline{X}\,\underline{8}\,\underline{5}$, when divided by 9, leaves a remainder of 2. What is the value of the obscured digit, X?

3. $ _____

[Source: MATHCOUNTS]

Terry plays a game with prizes of 5, 10, 15, and 20 dollars. The graph shows each possible prize amount and its corresponding probability. *Rounded to the nearest dollar*, what is the expected value of Terry's prize?

4. _____

In how many ways can the letters in BLUEFISH be rearranged so that no two vowels are adjacent?

5. _____

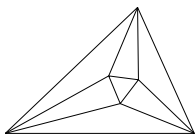Find the sum of all possible integer values of m for which the pair of lines

$$13x + 11y = 700$$
$$y = mx - 1$$

intersect at a lattice point (i.e. a point with integer coordinates).

Name: _____   Team: _____

1. _____ terms A geometric sequence has 20 terms. The sum of its first two terms is 40. The sum of its first three terms is 76. The sum of its first four terms is 130. How many of the terms in the sequence are integers?

2. $c^2 =$ _____ In triangle ABC, $\angle ABC = 15°$, $\angle BAC = 45°$, and $BC = 8$. If c is the length of $\overline{AB}$, find c^2.

3. _____ In going through her grandmother's papers, Salma found a calculation with all but one digit faded (shown below, where faded digits are designated by $\square$). Her grandmother loved math, and had noted that all digits were one- digit primes. What was the value of this product?

$$
\begin{array}{r}
\square\,\square \\
\times\ \square \\
\hline
3\,\square\,\square
\end{array}
$$

4. _____ $(a + b + c)^{15}$ is expanded and simplified. How many terms are in this simplified expression?

5. _____ What is the radius of the circle that goes through the points $A(6, 26)$, $B(24, 2)$, and $C(-16, -18)$?

6. _____ A *Heronian Triangle* is a triangle whose side lengths and area are all integers. For example, a 13-14-15 triangle with area 84 is a Heronian triangle.

If ABC is a non-isosceles Heronian triangle with height 20, determine the maximum possible area of $\triangle ABC$.

Team: _____
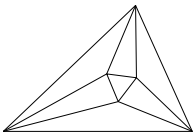
Minnesota State High School Mathematics League
2024-25 Sample Meet 4.2, Answers

Event A:

1. **18**
(20-21 SI6, 93% correct)

2. **108**
(20-21 4C1, 59% correct)

3. **12**
(20-21 SI11, 86% correct)

4. **3**
(20-21 3D2, 43% correct)

5. **49**
(21-22 3C4, 5.4% correct)

Event B:

1. **15**
(21-22 4D1, 81% correct)

2. **3**
(23-24 4B1, 57% correct)

3. **24**
(21-22 3C1, 75% correct)

4. **93**
(21-22 3B3, 22% correct)

5. **56**
(21-22 3T4, 34% correct)

Event C:

1. **3**
(23-34 5A2, 79% correct)

2. **6**
(08-09 5A1)

3. **13**
(18-19 MC Handbook Stretch 12)

4. **14400**
(22-23 5T4, 22% correct)

5. **6**
(20-21 2A4, 8.5% correct)

Team Event:

1. **2**
(21-22 ST2, 97% correct)

2. **96**
(23-24 3C2, 35% correct)

3. **375**
(23-24 5T2, 63% correct)

4. **136**
(20-21 5T3, 46% correct)

5. **25**
(23-24 2D3, 7.8% correct)

6. **1470**
(21-22 1T6, 11% correct)