

# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.1, Individual Event A

15 minutes

Score	Check

1.  $A + B =$  \_\_\_\_\_

In a  $3 \times 3$  Latin Square, every row and every column must contain each of the numbers 1, 2, and 3. Figure 1 shows a Latin Square with two cells filled. Find the value of  $A + B$ .

1		A
B		
	2	

Figure 1

2.  $A \times B \times C =$  \_\_\_\_\_

$A$ ,  $B$ , and  $C$  are positive real numbers for which  $A \times B = 20$ ,  $B \times C = 24$ , and  $A \times C = 30$ . Determine the value of  $A \times B \times C$ .

3.  $f(7) =$  \_\_\_\_\_

The function  $f$  is defined by  $f(n) = 3f(n - 1) - f(n - 2)$ , where  $n$  is any positive integer. If  $f(1) = 1$ , and  $f(2) = \frac{1}{3}$ , evaluate  $f(7)$ .

4. \_\_\_\_\_

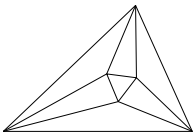
Let  $g(t) = 4 + 5t$  and  $h(t) = (g(t))^2$ . Determine exactly the value of  $h(g^{-1}(12))$ .

5.  $t_{20} =$  \_\_\_\_\_

Let  $t_n$  be the number of triples of integers  $(a, b, c)$ , where  $1 \leq a < c \leq n+1$  and  $1 \leq b < c < n+1$ . For instance  $t_2 = 5$ , with corresponding triples  $(1, 1, 2)$ ,  $(1, 1, 3)$ ,  $(1, 2, 3)$ ,  $(2, 1, 3)$ , and  $(2, 2, 3)$ . What is  $t_{20}$ ?

Name: \_\_\_\_\_

Team: \_\_\_\_\_



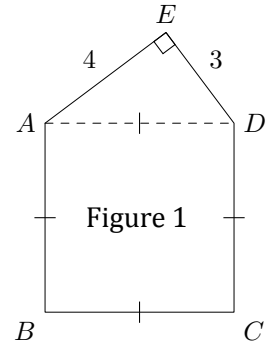
# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.1, Individual Event B

15 minutes

Score	Check

1. \_\_\_\_\_ Figure 1 shows pentagon  $ABCDE$ , formed by attaching the hypotenuse of right triangle  $AED$  to square  $ABCD$ . If  $AE = 4$  and  $DE = 3$ , find the area of pentagon  $ABCDE$ .



2. \_\_\_\_\_ % A  $10 \times 10 \times 10$  cube has its length increased by 10%, its width increased by 10%, and its height increased by 40%. Determine the percentage increase of its surface area.

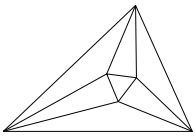
3.  $m + n =$  \_\_\_\_\_ The coordinates of the vertex of the graph of  $y = x^2 + 12x + 30$  are  $(m, n)$ . Determine the value of  $m + n$ .

4.  $m + n =$  \_\_\_\_\_ A pyramid's base is an equilateral triangle, and each face is an isosceles right triangle whose hypotenuse is an edge of the base. If the height of the pyramid is 2, the volume of the pyramid can be written in the form  $m\sqrt{n}$  where  $m$  and  $n$  are positive integers and  $n$  is square-free. Determine  $m + n$ .

5.  $d^2 =$  \_\_\_\_\_ For the complex number  $\omega = 2(\cos 20^\circ + i \sin 20^\circ)$ , define  $d = |\omega - \omega^4|$ . Find  $d^2$ .

Name: \_\_\_\_\_

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.1, Individual Event C

15 minutes

Score	Check

1.  $m + n =$  \_\_\_\_\_  
[Source: MATHCOUNTS]

The height of the shaded rectangle shown in Figure 1 is one-half the height of the square, and the width of the rectangle is one-third the width of the square. If a point is chosen at random inside the square, the probability that it is also inside the rectangle is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

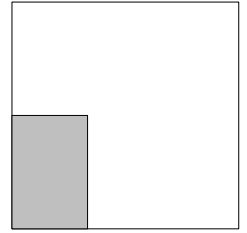


Figure 1

2. \_\_\_\_\_

Out of 135 students surveyed, 28 are able to juggle and 42 can do a handstand. If 88 students are unable to do either skill, how many students are able to juggle and can also do a handstand?

3. \_\_\_\_\_

How many ordered pairs of integers  $(x, y)$  satisfy the equation  $x^4 + y^2 = 4y$ ?

4.  $m + n =$  \_\_\_\_\_

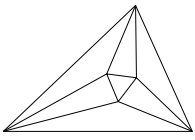
Sydney started the day with 15 coins in her pocket which totaled \$2.00. At the end of the day, after a number of transactions, she had 16 coins which totaled \$3.00. She had only quarters, dimes, nickels, and pennies, and ended the day with a different number of each type of coin than she had started with (one or the other of which could have been zero). If she started with  $m$  quarters and ended with  $n$  quarters, what is  $m + n$ ?

5.  $m + n =$  \_\_\_\_\_

The number  $23!$  has over 190 thousand positive integer divisors. If one of them is chosen at random, the probability that it is even is exactly  $\frac{m}{n}$  for some relatively prime positive integers  $m$  and  $n$ . Find  $m + n$ .

Name: \_\_\_\_\_

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.1, Team Event

30 minutes

Score	Check

1. \_\_\_\_\_ Three faces of a rectangular prism have perimeters 24, 26, and 30. What is its volume?

2.  $m + n =$  \_\_\_\_\_ If  $f(x) + f\left(\frac{1}{1-x}\right) = x^2$ , then  $f(2)$  can be written as  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .

3. \_\_\_\_\_ How many perfect square factors does 46,656 have?

[Source: MATHCOUNTS]

4. \_\_\_\_\_ Find the sum of the real solutions to  $x^3 - 1 = \sqrt{5x^3 + 9}$ .

5. \_\_\_\_\_ In the Venn diagram in Figure 5, find the sum of the numbers in the region:

$$\overline{A \cup B} \cap \overline{C \cup B} \cap \overline{A \cup C}.$$

(Notes:  $\overline{X}$ , the *complement* of set  $X$ , is everything NOT in set  $X$ .  $X \cup Y$ , the *union* of sets  $X$  and  $Y$ , is everything in set  $X$  or in set  $Y$  (or both), and  $X \cap Y$ , the *intersection* of  $X$  and  $Y$ , is everything which is in both sets  $X$  and  $Y$ .)

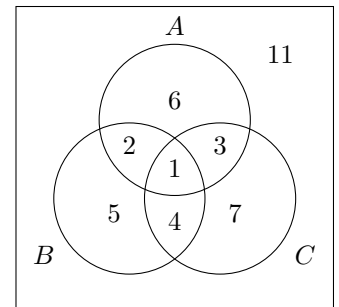


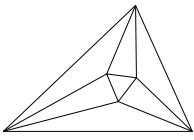
Figure 5

6. \_\_\_\_\_ The equation of a parabola with vertex  $(2, 4)$  and focus  $(5, 7)$  can be written in the form

$$x^2 + y^2 + axy + bx + cy + d = 0,$$

where  $a, b, c$ , and  $d$  are integers. Determine  $a + b + c + d$ .

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

## 2024-25 Sample Meet 5.1, Answers

### Event A:

1.

**4**

(23-24 SA1, 96% correct)

2.

**120**

(21-22 SA1, 96% correct)

3.

**-7**

(09-10 4A3)

4.

**144**

(20-21 SD1, 76% correct)

5.

**2870**

(22-23 4C4, 7.2% correct)

### Event B:

1.

**31**

(22-24 SB1, 92% correct)

2.

**43**

(23-24 SI8, 78% correct)

3.

**-12**

(20-21 4D1, 67% correct)

4.

**7**

(21-22 2T5, 30% correct)

5.

**228**

(23-24 3C4, 2.5% correct)

### Event C:

1.

**7**

(98-99 MATHOUNTS Probability Stretch 5)

2.

**23**

(21-22 5A2, 68% correct)

3.

**2**

(23-24 5C2, 36% correct)

4.

**18**

(22-23 3T5, 53% correct)

5.

**39**

(17-18 5D4, 4.0% correct)

### Team Event:

1.

**280**

(21-22 3T3, 86% correct)

2.

**21**

(22-23 SI12, 67% correct)

3.

**16**

(02-03 MATHCOUNTS Counting Stretch 9)

4.

**2**

(23-24 4T3, 40% correct)

5.

**5**

(17-18 5T2, 24% correct)

6.

**98**

(21-22 4T5, 8.8% correct)