

# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.2, Individual Event A

15 minutes

Score	Check

1.  $x =$  \_\_\_\_\_ Find the number  $x$  that satisfies  $(x - 1)(x - 8) = (x - 2)(x - 5)$ .

2.  $k =$  \_\_\_\_\_ If  $f(x) = \frac{k\sqrt{x+2}}{\sqrt{(3x-2)^3}}$ , and  $f(2) = 20$ , what is the value of  $k$ ?

3.  $G(2005) =$  \_\_\_\_\_  $G(1) = 3$  and  $G(n+1) = \frac{4G(n)+1}{4}$  for integers  $n > 1$ . Find  $G(2005)$ .

4. \_\_\_\_\_ Find the value of  $1^2 + 3^2 + 5^2 + 7^2 + \dots + 25^2 + 27^2 + 29^2$ , i.e. the sum of the squares of the first 15 odd numbers.

5.  $m + n =$  \_\_\_\_\_  $(m, n)$  are integers which satisfy the system of equations:

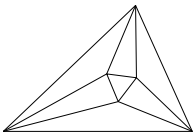
$$m^2 - 7m - 120 = 35n$$

$$m^2 - 9n^2 = 2m - 6n$$

Determine the value of  $m + n$ .

Name: \_\_\_\_\_

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.2, Individual Event B

15 minutes

Score	Check

1.  $k =$  \_\_\_\_\_ (1, 11) and (7,  $k$ ) are both points on the graph of the parabola  $y = 2(x - 4)^2 - 7$ , where  $k$  is an integer. Find  $k$ .

2. \_\_\_\_\_  $\text{ft}^3$  A tent is shaped like a triangular prism, with a floor measuring 8 ft wide by 10 ft long and a height of 7 ft, as shown in Figure 2. Determine the interior volume of the tent.

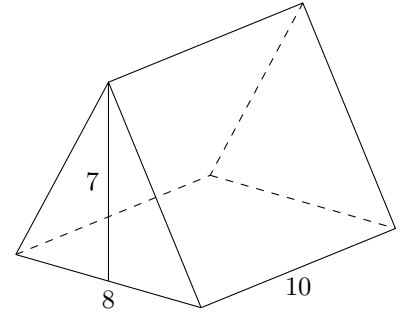


Figure 2

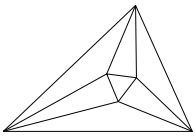
3. \_\_\_\_\_  $\text{miles}^2$  On a map, Minnesota has an area of 100 square inches. If the scale of the map is 1 inch : 30 miles, what is the actual square mileage of Minnesota?

4.  $AB =$  \_\_\_\_\_ Consider two concentric circles of radius 14 and 22. A chord  $\overline{AB}$  of the larger circle passes through the smaller circle, forming chord  $\overline{CD}$ . If  $AB = 3 \cdot CD$ , what is the length of  $\overline{AB}$ ?

5. \_\_\_\_\_ Let  $c$  be a positive integer. There are values of  $c$  for which the complex roots of  $z^2 - cz + 17$  can be written as  $a \pm bi$  for positive integers  $a$  and  $b$ . Graphing just those solutions for  $z$  in the complex plane form the vertices of a polygon. What is the area of this polygon?

Name: \_\_\_\_\_

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.2, Individual Event C

15 minutes

Score	Check

1.  $N =$  \_\_\_\_\_ Let  $N$  be the 4-digit number  $\underline{a}\underline{b}\underline{c}\underline{d}$ , where  $a^2 + d^2 = 13$  and  $b^2 + c^2 = 85$ . Given that  $N - 1089 = \underline{d}\underline{c}\underline{b}\underline{a}$ , the 4-digit number with the digits of  $N$  reversed, find  $N$ .

2. \_\_\_\_\_ % Pierre throws darts that land randomly in the dartboard shown in Figure 2. The dartboard is a circle of radius 2 units, with an inner circle of radius 1 unit. Both circles are divided into six congruent sectors. What is the probability that a dart Pierre throws will land in one of the four inner numbered sectors? Express your answer as a percentage, rounded to the nearest integer.

[Source: MATHCOUNTS]

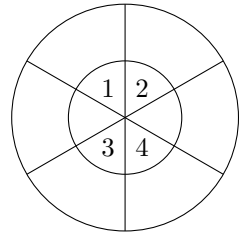


Figure 2

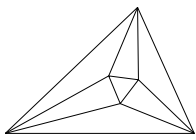
3.  $D =$  \_\_\_\_\_ When 9638, 8739, and 2591 are divided by integer  $D$ , where  $D > 1$ , the remainder is the same for all three divisors. Determine the value of  $D$ .

4. \_\_\_\_\_ days My cat Delta meows, hisses, and purrs. I heard her make at least one of these sounds on each of the past 30 days. In these 30 days, she hissed on 8 of the days, purred on 14 of the days, and meowed on 18 of the days. On 3 of the days, I heard her meow and hiss but not purr, and on 3 of the days, I heard her purr and hiss but not meow. On one day, she made all three sounds! On how many of these 30 days did I hear her meow and purr but not hiss?

5. \_\_\_\_\_ For how many integer bases  $b \geq 2$  does the base  $b$  representation of 2021 end in the base  $b$  digit 1? For instance,  $b = 5$  is one such example, since  $2021 = 31041_5$ .

Name: \_\_\_\_\_

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

2024-25 Sample Meet 5.2, Team Event

30 minutes

Score	Check

1.  $k =$  \_\_\_\_\_ When  $y = x^2 - 8x + 12$  is graphed, its axis of symmetry is the line  $x = k$ . Find  $k$ .

2.  $m + n =$  \_\_\_\_\_ A point is selected in the region bounded by  $|x - 1| < 2$ ,  $y < -x + 1$ , and  $y \geq -2$ . The probability that the point is in quadrant IV is  $\frac{m}{n}$ , where  $m$  and  $n$  are relatively prime positive integers. Find  $m + n$ .  
[Source: MATHCOUNTS]

3.  $k =$  \_\_\_\_\_ Define  $f(x) = \log\left(\frac{1+x}{1-x}\right)$ , and  $g(x) = f\left(\frac{3x+x^3}{1+3x^2}\right)$ . Then  $g(x)$  can be simplified so that  $g(x) = k \cdot f(x)$ . Determine the value of  $k$ .

4. \_\_\_\_\_ Complete the cross-number puzzle in Figure 4 by putting the proper digit into each box. All answers to the clues are three-digit numbers and no answer begins with 0. What is the sum of all nine digits?

*Across:*

- The smallest three-digit Fibonacci number that is also a perfect square.
- A perfect square whose digits are in increasing order from left to right.
- A number that is the product of three primes.

*Down:*

- A number divisible by 14.
- The value of  $4_{11} \cdot 100_{11}$  in base 10.
- A triangular number.

	1	2	3
1			
2			
3			

Figure 4

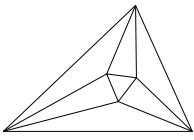
5.  $m + n =$  \_\_\_\_\_ A hexagonal pyramid has as its base a regular hexagon of side length 8, and its top vertex is equidistant to each vertex of the base. Given that the pyramid's surface area is  $432\sqrt{3}$ , its volume can be written in the form  $m\sqrt{n}$  where  $m$  and  $n$  are positive integers and  $n$  is square-free. Find  $m + n$ .

6. \_\_\_\_\_  $\{a_1, a_2, a_3, \dots\}$  is a sequence defined recursively by:

$$a_n = \begin{cases} 1 & \text{if } n = 1, \\ a_{n-1} + n \cdot 2^{n-1} & \text{if } n > 1. \end{cases}$$

When  $a_{2022}$  is written in binary (i.e. as a base 2 number), how many of its digits are 1?

Team: \_\_\_\_\_



# Minnesota State High School Mathematics League

## 2024-25 Sample Meet 5.2, Answers

### Event A:

1.

**-1**

(23-24 2T1, 99% correct)

2.

**80**

(20-21 4A1, 67% correct)

3.

**504**

(09-10 4A3)

4.

**4495**

(23-24 4C3, 30% correct)

5.

**24**

(21-22 SA4, 18% correct)

### Event B:

1.

**11**

(23-24 4D1, 86% correct)

2.

**280**

(21-22 2B2, 69% correct)

3.

**90000**

(22-23 SB1, 67% correct)

4.

**36**

(21-22 5D3, 14% correct)

5.

**15**

(22-23 5D4, 7.8% correct)

### Event C:

1.

**3762**

(20-21 1T1, 84% correct)

2.

**17**

(17-18 MATHCOUNTS Prob. Stretch 10)

3.

**29**

(18-19 ST4, 97% correct)

4.

**2**

(18-19 5A3, 28% correct)

5.

**11**

(21-22 1T4, 31% correct)

### Team Event:

1.

**4**

(21-22 ST2, 93% correct)

2.

**3**

(00-01 MATHCOUNTS Probability Stretch 7)

3.

**3**

(23-24 3C2, 60% correct)

4.

**44**

(23-24 5T6, 36% correct)

5.

**581**

(23-24 3T3, 19% correct)

6.

**9**

(21-22 4T6, 10% correct)