



Preparing for Change: The 2024-25 Math League Season

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- Head of the Problem Writing Team

Our Problem Writing Team



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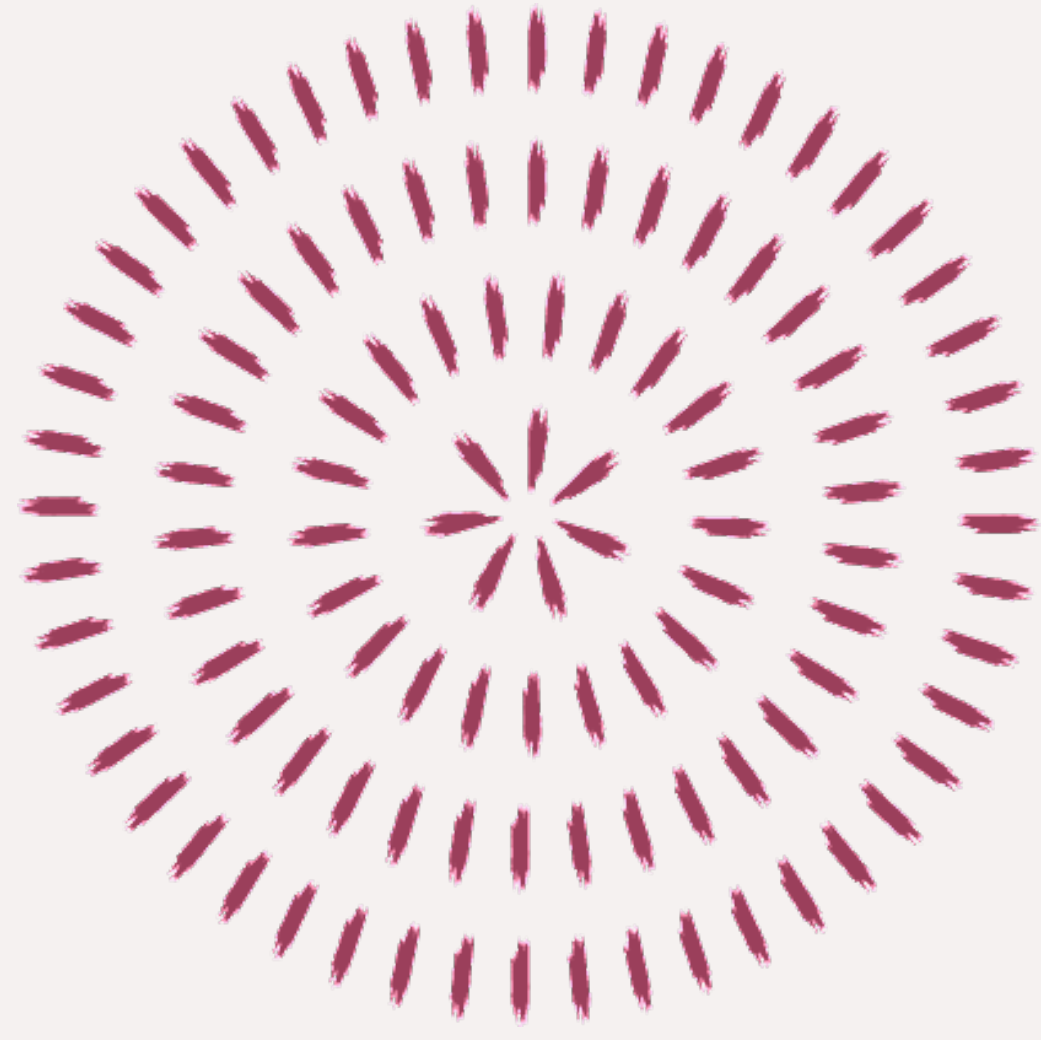
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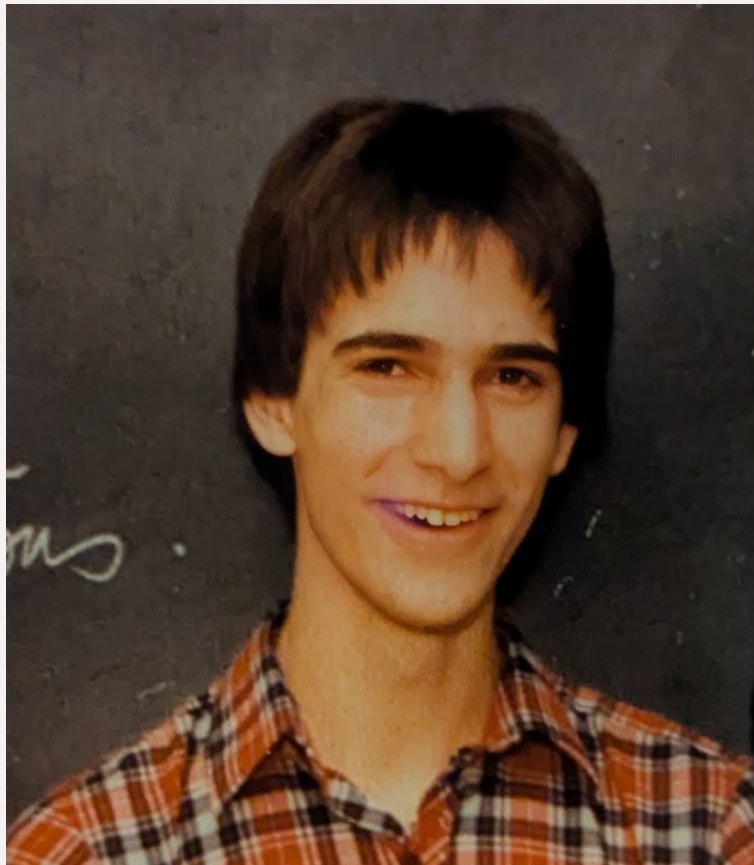
My math competition story ...





**Grade 12: my
proudest high school
achievement**

Grade 10: my first math competition experience:



PART A: You have 10 minutes to complete as many questions in this section as you can.

1. If $A = 2B$ and $B = 4C$, then calculate the value of $A + 2B - 8C$. (Express the answer in terms of C only.)

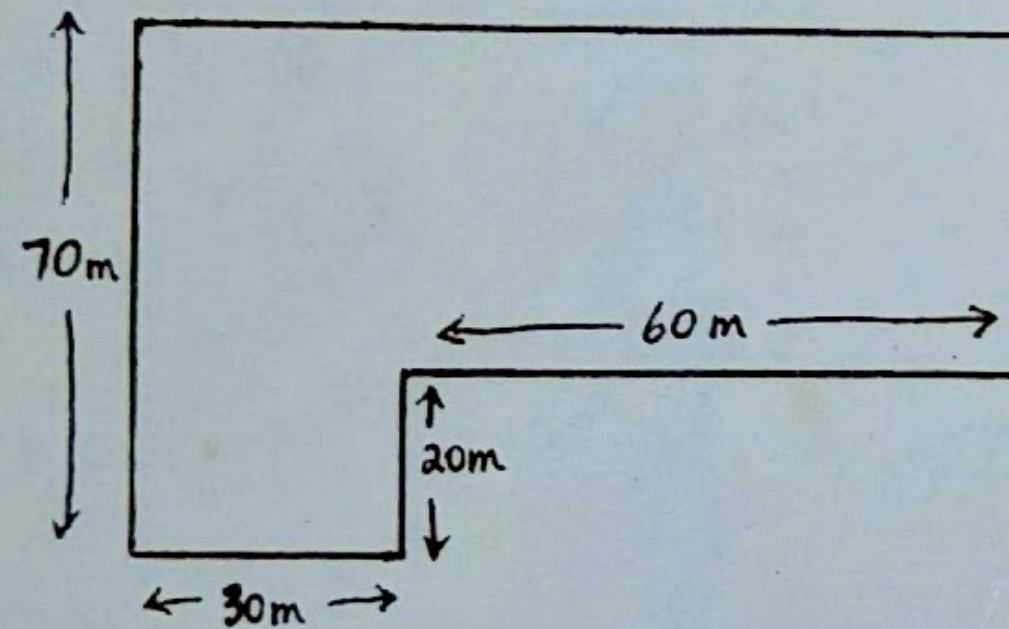
8C ✓

2. If $a:b = 3:4$ and $a:(b+c) = 2:5$, calculate the ratio $a:c$.

2:7 X

3. George has a lawn with dimensions as shown. If he has mowed a 10m strip around the outside, what fraction of the lawn has been mowed?

2800 X



4. A pool filter removes half of the dirt in a pool in 2 hours. To the nearest hour, find the time required to remove 99.9% of the dirt in the pool.

4 hours X

Name: Colin Springer

Grade: 10

Thank you teachers and coaches!

- Your work is exceptionally important ...
- ... and thankless and hard ...
- None of this would be possible without you.
- You are my heroes!

Plan for my three sessions:

Now: 2024-25 Event Changes

This afternoon: Sample Meet

Tomorrow: Coach Resources

Goals as I create Events



Interesting
Problems

Unique
Problems

Understandable
Problems

Nearly everyone
can solve one

Very few can
solve all

Feel they could
have solved
more

Develop
Problem Solving
Skills

Pave the way for
future success

I need your feedback!

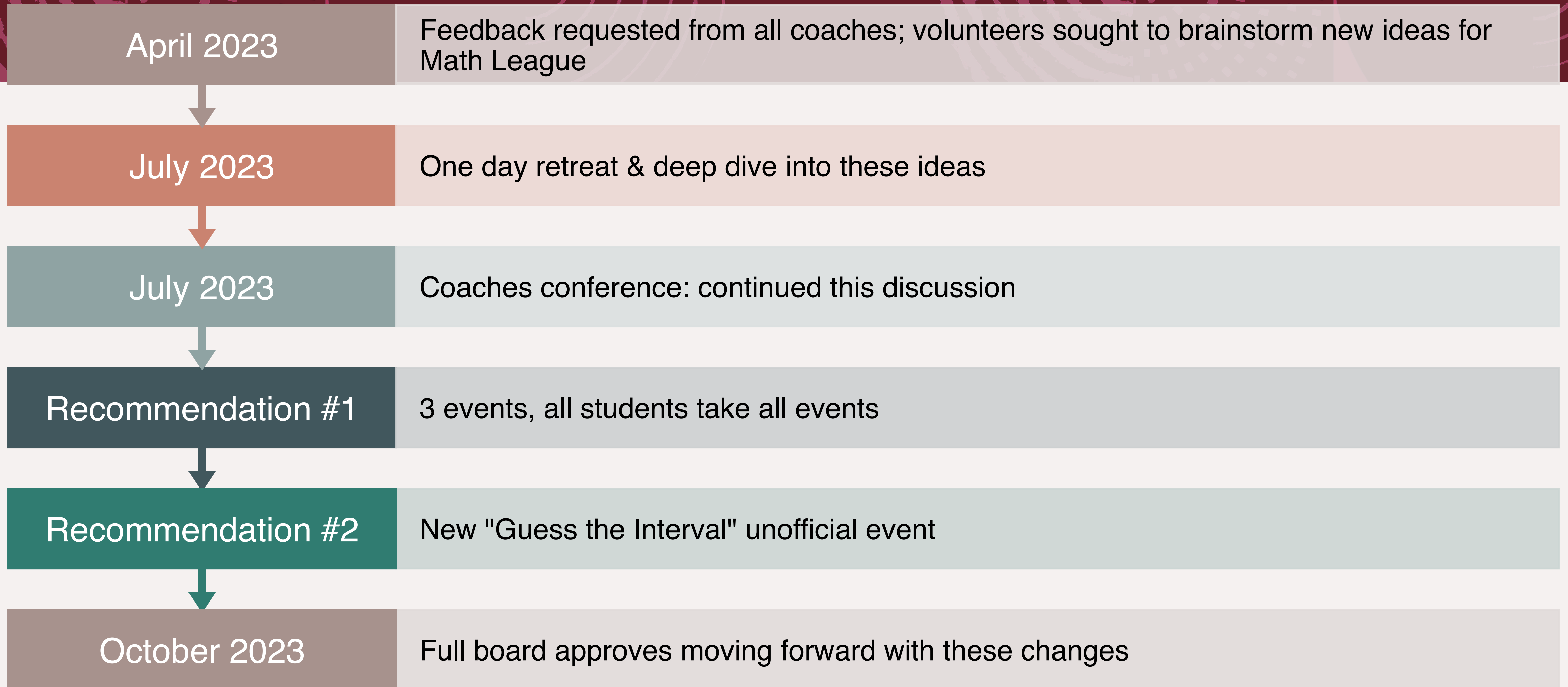
- You have visibility to student reception; I don't
- Tell me when my goals aren't (or are) achieved

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How these format changes came about:



How we got here (continued)

Work on three event
realignment
Formula Sheet proposal
Executive Committee
discussions

Full board reviews
feedback, approves
proposal

Oct. 2023 –
Feb. 2024

Mar. 2024

Apr. 2024

Proposals sent to all,
with feedback survey
request

Where things stand:

New format is happening for 2024-25

Feedback survey to be sent at season end

Full board may decide on adjustments from 2025-26 onward

Three events, all students take them all

Easier to coach

No hard decisions
about event
assignments

Encourages more
well rounded
mathematical
studies

Allows for more
accessible
problems

Individual scores
more comparable

Year over year
growth more
visible

The Big Challenge:

Less experienced students will face topics they're unfamiliar with

- It's on me to make problems reasonably accessible
- I need you to help set reasonable expectations

Individual Event Logistics:

Five problems per event

1 point per problem (5 per event)

15 minutes per event

Survey: How hard should individual problems be?



bit.ly/4eYtadL

Individual Problem Difficulty

Approximately what percentage of students (all grades, statewide) do you believe should correctly solve each problem?

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✉ Not shared

☁

* Indicates required question

Problem 1 (easiest) *

Your answer _____

Problem 2 *

Your answer _____

My (Approximate) Individual Targets:

- Problem 1: 90% (and solvable very quickly)
- Problem 2: 80% (and solvable very quickly)
- Problem 3: 50% (also a fairly quick solve)
- Problem 4: 25%
- Problem 5: 1% to 10%

... but I generally overestimate student performance

Your Survey Responses:

	Median Response
Problem 1:	90
Problem 2:	75
Problem 3:	50
Problem 4:	30
Problem 5:	10

1 Point per Problem

- Lessens score gap
 - Currently a student with 1 correct problem out of 4 gets only 14% of possible event points
- Power scoring will continue to be used for ranking purposes

Overall Scoring Changes:

- 5 points possible per individual event, 15 per individual meet (vs. 14 today)
- Team round problems increased to 5 points each, for 30 possible points (vs. 24 today)
- Overall maximum team score of 150 per event (vs. 136 today)



New Event Tracks:

A: Algebra

B: Geometry & Trigonometry

C: Counting, Probability & Statistics,
Number Theory

2024-25 MSHSML Topics

A - Algebra	B - Geometry & Trigonometry	C - Counting, Probability & Statistics, Number Theory
1A 1. Decimals, Fractions, and Percents 2. One Variable Linear Equations & Inequalities 3. Exponent Rules 4. Square Roots and Radicals	1B 1. Angles & Angle Relationships 2. Triangle Similarity and Congruence 3. Analytic Geometry of a Straight Line 4. Trigonometry Basics	1C 1. Basic Counting 2. Statistical Measures 3. Prime Factorization & Divisibility Rules 4. GCD and LCM
2A 1. Systems of Linear Equations 2. Binomials and Quadratics 3. Absolute Value 4. The Logarithm	2B 1. Area, Perimeter, and Lengths in Triangles 2. Right Triangles 3. Analytic Geometry of Points and Lines 4. More Elementary Trigonometry	2C 1. Counting Permutations and Independent Events 2. Analyzing Data 3. Basic Probability 4. Base n Arithmetic
3A 1. The Quadratic Formula 2. Polynomials 3. Arithmetic Sequences & Series 4. Complex Number Arithmetic	3B 1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons 2. Problem Solving involving Triangles, Quadrilaterals and Polygons 3. Transformations in the Plane 4. Trigonometric Identities	3C 1. Counting Combinations 2. More Probability 3. Pascal's Triangle and the Binomial Theorem 4. Calculating Digits, especially the Last Digit
4A 1. Optimization Problems 2. Roots of Polynomial Equations 3. Geometric Sequences & Series 4. More Logarithms & Exponents	4B 1. Area, Perimeter, Angles in Circles 2. Lengths involving Circles 3. Analytic Geometry of Circles 4. Law of Sines & Law of Cosines	4C 1. More Counting & Probability 2. Expected Value 3. Remainders & Modular Arithmetic 4. Finding Integer Solutions
5A 1. Non-Linear Equations and Systems of Equations 2. Functional Equations 3. Sums of Powers of Integers 4. General Sequences & Series	5B 1. 3-Dimensional Geometry 2. Geometry Problem Solving 3. Analytic Geometry of Conic Sections 4. Geometry of Complex Numbers	5C 1. Divisor Arithmetic 2. Venn Diagrams & the Principle of Inclusion-Exclusion 3. Geometric Probability 4. More Integer Solutions

Notes:

- Problems may draw on topics from any previous meet.
- Logic or puzzle type problems may appear in any event throughout the season.

Event A

- Includes most topics from prior A and D events
- Shuffled to ensure some accessible topics each meet
- Advanced topics more likely to appear as #5 or on team round

Event B

- Includes most topics from prior B and C events
- Trigonometry de-emphasized (but still present)

Event C

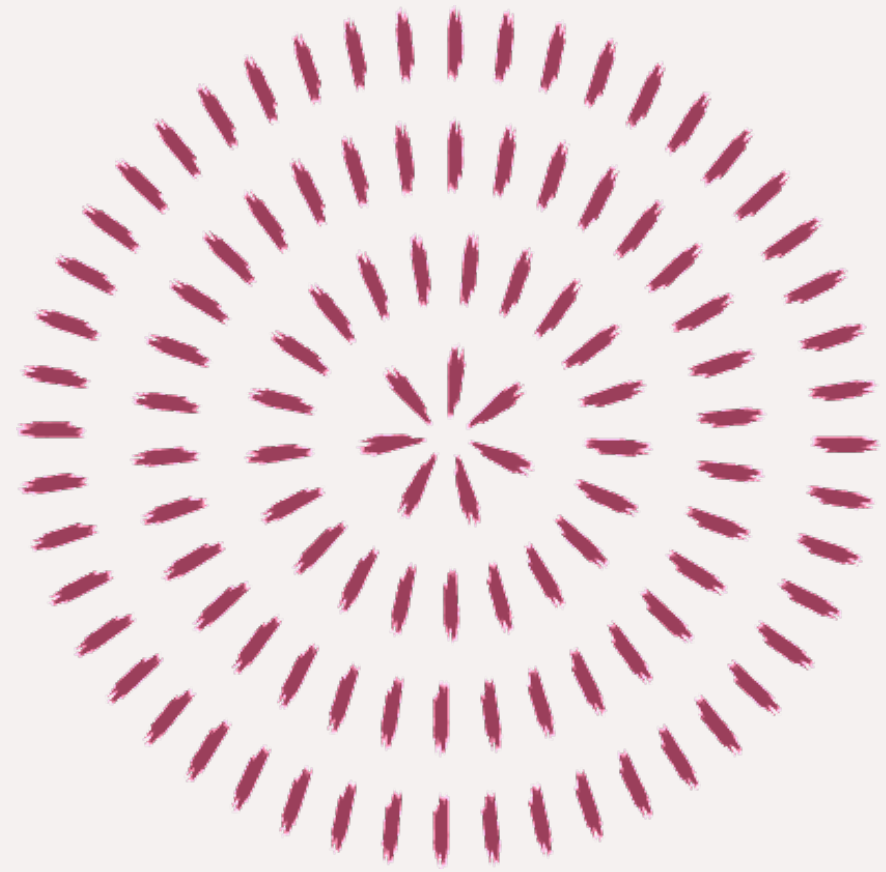
- Counting, Probability & Statistics, Number Theory expanded into new C track
- Balance better aligns with AMC 10/12, etc.
- Topics lend themselves to understandable problems

Puzzle & Logic Problems (former 5A)

- No separate event for Puzzle / Logic problems
- Such problems will still be seen throughout the season

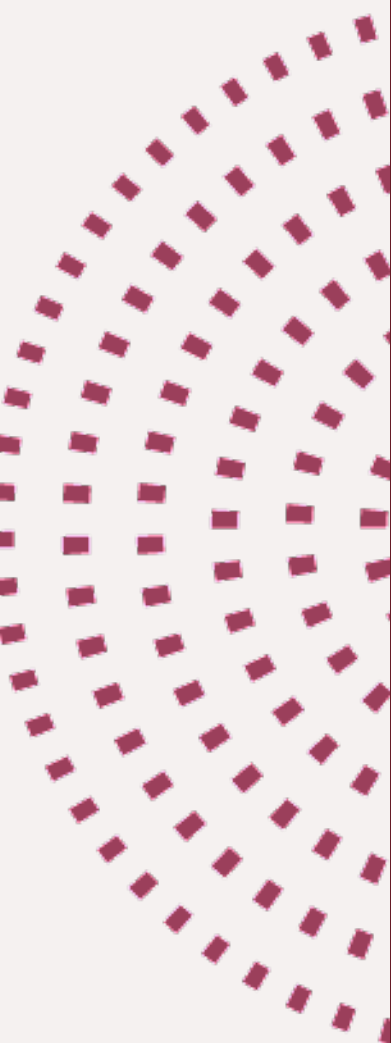
AMC 12 Problems (former 5D)

- No dedicated AMC preparation event
- But ...



HINT!!!

- For 2024-25, at least one problem on Meet 1 is nearly identical to one of the first ten problems from 2023's AMC 12A



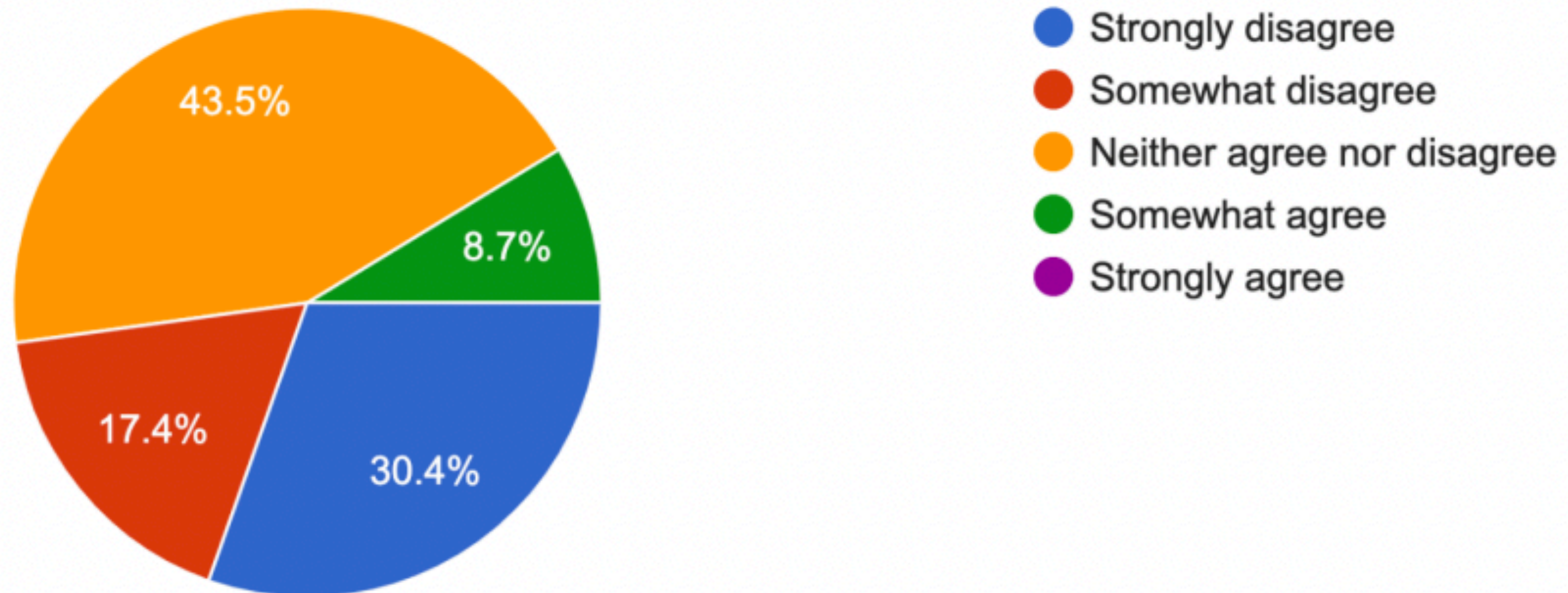
Enabling Success for Less Experienced Students

- We want Understandable, Accessible problems
- This is now much harder!

Why not separate topic lists?

7. Separate 9th/10th and 11th/12th grade topics (and tests) should be offered (similar to AMC 10 vs AMC 12).

23 responses



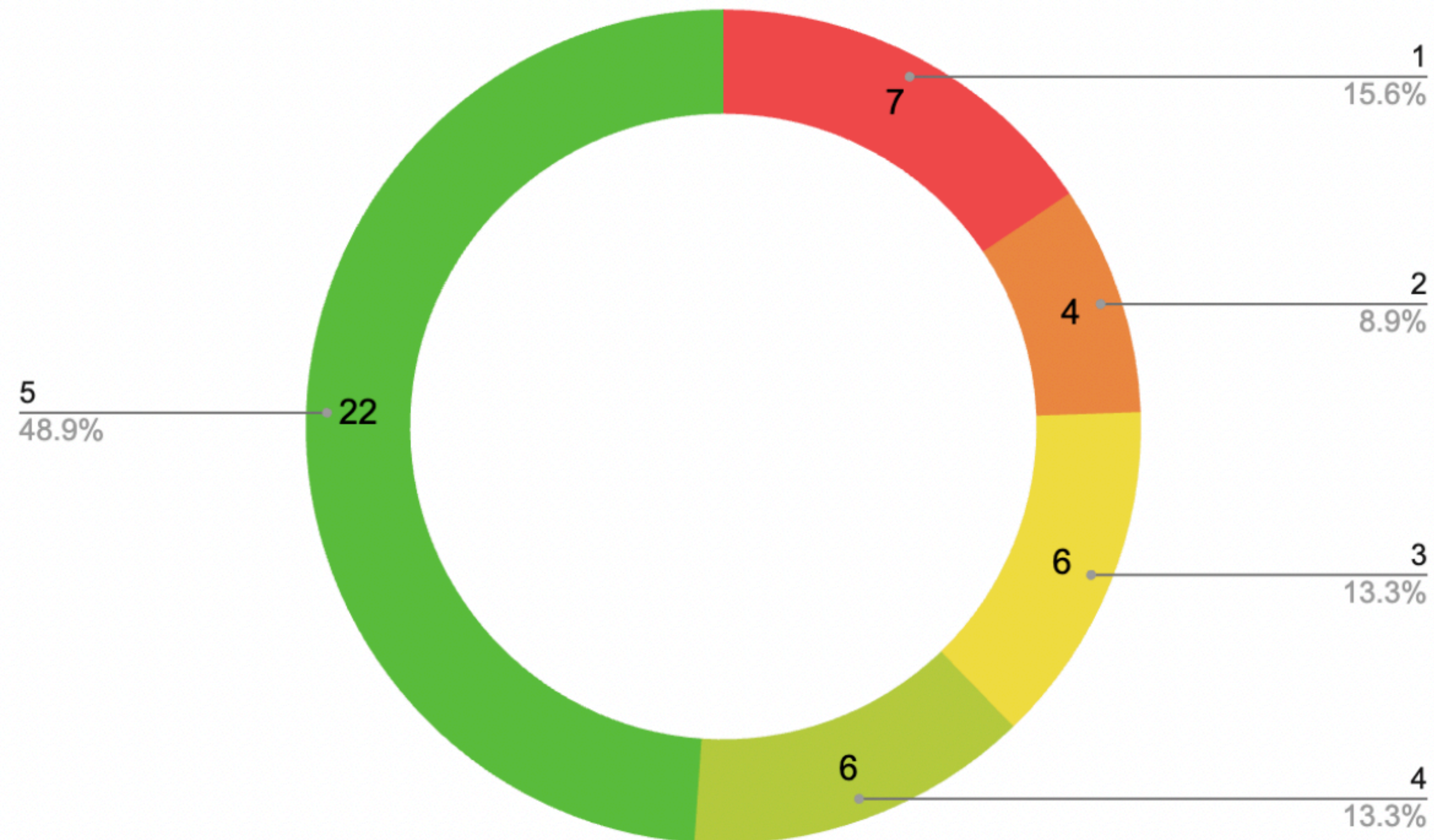
Trying to level the playing field:

- Starting this season, students will have a standardized formula sheet for reference during meets

Standardized Formula Sheet

- This was somewhat controversial!

Students should be provided with a formula sheet, that is created by the League, to use during the Meets.



The MSHSML Formula Sheet

ANGLE BISECTOR THEOREM:

$$\frac{b}{c} = \frac{m}{n}$$

CEVA'S THEOREM:

$$\frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} = 1$$

MENELAUS' THEOREM:

$$\left| \frac{AY}{YC} \cdot \frac{CX}{XB} \cdot \frac{BZ}{ZA} \right| = 1$$

CYCLIC QUADRILATERALS:

ABCD is Cyclic

$$\Leftrightarrow \angle ADB = \angle ACB$$

$$\Leftrightarrow \angle ABC + \angle ADC = 180^\circ$$

Inscribed Angle Theorem:

$$\angle AOB = 2\angle ADB$$

Ptolemy's Theorem: $ac + bd = AC \cdot BD$

Brahmagupta's Formula:

$$\text{Area} = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \quad \left(s = \frac{a+b+c+d}{2} \right)$$

STEWART'S THEOREM:

$$b^2m + c^2n = a(d^2 + mn)$$

MORE TRIANGLE RESULTS:

Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's Formula:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \quad \left(s = \frac{a+b+c}{2} \right)$$

LOGARITHMS:

$y = \log_b x$ means $b^y = x$

$$\log_n(ab) = \log_n a + \log_n b$$

$$\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$$

$$\log_n(a^k) = k \log_n a$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

QUADRATIC FORMULA:

Roots of $ax^2 + bx + c = 0$

$$\text{are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

AM-GM INEQUALITY:

If $a_i \geq 0$, then

$$\frac{a_1 + a_2 + \dots + a_n}{n} \geq \sqrt[n]{a_1 a_2 \dots a_n}$$

SUM AND DIFFERENCE OF POWERS:

$$a^2 - b^2 = (a-b)(a+b)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

SUMS OF POWERS OF INTEGERS:

$$1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$$

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$$

ANALYTIC GEOMETRY OF LINES:

- The line joining (x_1, y_1) to (x_2, y_2) has slope $\frac{y_2 - y_1}{x_2 - x_1}$
- The equation of line through (x_1, y_1) with slope m is $y - y_1 = m(x - x_1)$
- The distance from (x_0, y_0) to the line $ax + by + c = 0$ is $\frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$

SOME 3D GEOMETRY:

- Pyramids and cones with base area B and height h have volume $V = \frac{1}{3}Bh$.
- Volume of a sphere is $V = \frac{4}{3}\pi r^3$
- Surface area of a sphere is $SA = 4\pi r^2$

COUNTING OBJECTS:

Given n distinguishable objects:

- The # of ways to line them up in a row is $n! = n \cdot (n-1) \cdot \dots \cdot 3 \cdot 2 \cdot 1$
- The # of ways to pick a set of r where order matters is ${}^n P_r = \frac{n!}{(n-r)!}$
- The # of ways to pick a set of r where order doesn't matter is $\binom{n}{r} = \frac{n!}{r!(n-r)!}$

BINOMIAL THEOREM:

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

COMPLEX NUMBERS:

Define $i = \sqrt{-1}$, so $i^2 = -1$. Let $z = a + bi$.

Then $\bar{z} = a - bi$ is the complex conjugate of z , and $|z| = \sqrt{a^2 + b^2}$ is the modulus of z .

De Moivre's Theorem:

If $z = r(\cos \theta + i \sin \theta)$, then $z^n = r^n(\cos(n\theta) + i \sin(n\theta))$

ANALYTIC GEOMETRY OF CONIC SECTIONS:

- A **parabola** is the set of points equidistant from a point (the focus) and a line (the directrix)
- A parabola with vertex $(0, 0)$, focus $(0, p)$ and directrix $y = -p$ has equation $y = 4px^2$.
- An **ellipse** is the set of points whose distances to two fixed points (its foci) add to a constant.
- An ellipse centered at $(0, 0)$ with vertices $(\pm a, 0)$ and **covertices** $(0, \pm b)$ has area πab and equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 - b^2$.
- A **hyperbola** is the set of points whose distances to two points (its foci) have a constant difference.
- A hyperbola centered at $(0, 0)$ with vertices $(\pm a, 0)$ and asymptotes $y = \pm \frac{b}{a}x$ has equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Its foci are at $(\pm c, 0)$, where $c^2 = a^2 + b^2$.

TRIGONOMETRY DEFINITIONS:

Right triangle (SOH-CAH-TOA) & Unit Circle definitions:

$$\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}}, \quad \cos \theta = \frac{\text{adjacent}}{\text{hypotenuse}}, \quad \tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Graphs: (π radians = 180°)

SOME TRIGONOMETRIC IDENTITIES:

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \quad \sin^2 \theta + \cos^2 \theta = 1$$

$$\cot \theta = \frac{1}{\tan \theta}, \quad \sec \theta = \frac{1}{\cos \theta}, \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \frac{\sin A}{1 + \cos A} = \frac{1 - \cos A}{\sin A}$$

2024-25 Formula Sheet

Standardized Formula Sheet

- Goal is to make more problems accessible to newer students
- Many of these should already be known
- Goal is to encourage preparation through problem solving (not memorization)
- One sheet covers the entire season
- Up for review at season completion



My Transition Goals


1. Math League should feel familiar

2. Interested students should find success in Math League

3. Top students should continue to be challenged



My Requests of You

- Keep an open mind with this year's changes
 - Remind students that the problems are hard - celebrate any success!
 - Encourage all interested students to participate
 - Remind them that you can be good at math without being good at contests
 - Make practice materials available to students
 - Encourage learning from mistakes - share solution packets
 - Encourage top students to join the MN All State Math Team
 - Encourage neighboring schools to participate
 - Encourage feeder schools to offer MATHCOUNTS, AMC 8, MNJHML
 - Share ways I can improve MSHSML meets!
- 

Digging into the topics

- Previously it wasn't always clear what a topic included
- E.g. Trig reduction formulas, normal form of a straight line, sums of functions of angles
- New detailed breakdowns of topics covered in each event

Please note!

- This level of detail can seem overwhelming
- Nearly any problem in the "new" topics could have appeared before
- More advanced topics are likely to appear as Individual #5 or on the Team round
- Probably shouldn't teach every topic yearly

Meet 1 Topics Breakdown

1A. Algebra

1A.1. Decimals, Fractions, and Percents

- Adding, subtracting, multiplying, or dividing fractions and decimals
- Reducing fractions to lowest terms
- Converting fractions to decimals
- Converting decimals (terminating or repeating) to fractions
- Percent increase & decrease
- Ratios & Proportions
- Interest problems

1A.2. One Variable Linear Equations & Inequalities

- Solving linear equations in one variable
- One variable linear inequalities
- Word problems leading to linear equations or inequalities
- Rate problems (distance = rate \times time)
- Unit Conversion

1A.3. Exponent Rules

- Positive integer exponents
- $a^m a^n = a^{m+n}$, $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-n} = \frac{1}{a^n}$, $a^0 = 1$

1A.4. Square Roots & Radicals

- Square roots
- Cube roots & higher roots
- Simplifying square roots & radicals of integers
- Adding, subtracting, multiplying, and dividing radicals
- Rationalizing simple radical denominators
- Radicals as exponents: $\sqrt[n]{a} = a^{\frac{1}{n}}$

1B. Geometry & Trigonometry

1B.1. Angles & Angle Relationships

- Angle sums in triangles and polygons
- Parallel lines and angle relationships

1B.2. Triangle Similarity and Congruence

- Conditions for congruence: SSS, SAS, ASA, AAS
- Conditions for similarity: AA, SAS, SSS
- Problem solving in triangles using Similarity and Congruence

1B.3. Analytic Geometry of a Straight Line

- Slope
- Slope-intercept form of a straight line
- Point-slope form of a straight line
- Midpoint of a segment

1B.4. Trigonometry Basics

- Degrees, radians, and converting between the two
- Right Triangle trig definitions
- Unit Circle trig definitions

1C. Counting, Prob. & Stats, Num. Theory

1C.1. Basic Counting

- Counting lists
- Counting by cases
- Complementary counting

1C.2. Statistical Measures

- Average/mean
- Median
- Mode

1C.3. Prime Factorization & Divisibility Rules

- Testing for divisibility by 2 through 12 (except 7)
- Prime factorization (using divisibility rules and testing primes up to square root)

1C.4. GCD and LCM

- Calculating the Greatest Common Divisor by factoring
- Calculating the Least Common Multiple by factoring
- Calculating Greatest Common Divisor using the Euclidean Algorithm
- $\gcd(a, b) \times \text{lcm}(a, b) = ab$

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- $(a^m)^n = a^{mn}$

- $a^{-n} = \frac{1}{a^n}, a^0 = 1$

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- Conditions for similarity: AA, SAS, SSS
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- Calculating the Least Common Multiple by factoring
- Calculating Greatest Common Divisor using the Euclidean Algorithm
- $\text{gcd}(a, b) \times \text{lcm}(a, b) = ab$

Meet 2 Topics Breakdown

2A. Algebra

2A.1. Systems of Linear Equations

- Two (or occasionally more) variable linear systems of equations
- Word problems leading to systems of equations

2A.2. Binomials and Quadratics

- Multiplying binomials
- Rationalizing denominators using conjugates
- Difference of squares factorization
- Factoring a quadratic as a product of binomials
- Solving quadratic equations by factoring

2A.3. Absolute Value

- Solving absolute value equations and inequalities in one variable
- Representation on the Number Line

2A.4. The Logarithm

- Definition of logarithm
- Relationship to exponents
- $\log_n(ab) = \log_n a + \log_n b$
- $\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$
- $\log_n(a^k) = k \log_n a$

2B. Geometry & Trigonometry

2B.1. Area, Perimeter, and Lengths in triangles

- Base-height area formula
- Medians, angle bisectors, altitudes
- Triangle Inequality
- Heron's Formula for triangle area
- Triangle theorems: Angle Bisector, Stewart, Ceva, Menelaus

2B.2. Right Triangles

- The Pythagorean theorem
- 30-60-90 and 45-45-90 triangles
- Common Pythagorean Triples

2B.3. Analytic Geometry of Points and Lines

- Distance between points
- Finding intersection points of lines
- Systems of inequalities used to define a region in the plane
- Areas of polygons on a grid
- Distance from a point to a line

2B.4. More Elementary Trigonometry

- Trig functions of common angles
- Basic identities: $\sin^2 \theta + \cos^2 \theta = 1$,
 $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$,
 $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$, etc.

- Graphs of trig functions (sin, cos, tan)
- Inverse trigonometric functions
- Triangle area using trigonometry:
$$A = \frac{1}{2}ab \sin C$$
- Solving trigonometric equations

2C. Counting, Probability & Statistics, Number Theory

2C.1. Counting Permutations and Independent Events

- The multiplication principle for counting
- Counting Permutations: ${}_n P_r$
- Factorials
- Knowing when to add and when to multiply

2C.2. Analyzing Data

- Analyzing data in tables
- Analyzing data in charts
- Analyzing data in graphs

2C.3. Basic Probability

- Definition of probability
- Calculating probabilities using basic counting & permutations

2C.4. Base n Arithmetic

- Base n numbers and base n arithmetic
- Converting base n numbers to and from base 10
- Converting base n numbers to and from other bases

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- $\log_n \left(\frac{a}{b} \right) = \log_n a - \log_n b$
- $\log_n (a^k) = k \log_n a$

2B.1. Area, Perimeter, and Lengths in triangles

- Base-height area formula
- Medians, angle bisectors, altitudes
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- 30-60-90 and 45-45-90 triangles
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2B.4. More Elementary Trigonometry

- Trig functions of common angles

- Basic identities: $\sin^2 \theta + \cos^2 \theta = 1$, $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$,

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta \right), \text{ etc.}$$

- Graphs of trig functions (sin, cos, tan)

- Inverse trigonometric functions

- Triangle area using trigonometry: $A = \frac{1}{2}ab \sin C$

- Solving trigonometric equations

2C.1. Counting Permutations and Independent Events

- The multiplication principle for counting
- Counting Permutations: ${}_n P_r$
- Factorials
- Knowing when to add and when to multiply

2C.2. Analyzing Data

- Analyzing data in tables
- Analyzing data in charts
- Analyzing data in graphs

2C.3. Basic Probability

- Definition of probability
- Calculating probabilities using basic counting & permutations

2C.4. Base n Arithmetic

- Base n numbers and base n arithmetic
- Converting base n numbers to and from base 10
- Converting base n numbers to and from other bases

Meet 3 Topics Breakdown

3A. Algebra

3A.1. The Quadratic Formula

- Solving quadratics by completing the square
- Using the Quadratic Formula to solve quadratic equations
- The discriminant and character of roots
- Quadratic Inequalities

3A.2. Polynomials

- Multiplying polynomials
- Finding integer or rational roots of polynomials (the Rational Root Theorem)
- Factoring polynomials based on a known root; The Remainder Theorem
- Sum and difference of cubes factorization
- Sum and difference of odd powers factorization
- Solving polynomial equations
- Simplifying rational expressions (including Polynomial Division)
- Solving rational equations

3A.3. Arithmetic Sequences and Series

- Arithmetic sequence definition
- Finding the common difference
- Finding the n th term
- Arithmetic series definition
- Calculating the sum of an arithmetic series
- Arithmetic sequence & series problem solving

3A.4. Complex Number Arithmetic

- Adding and Subtracting complex numbers
- Multiplying complex numbers
- The Complex Conjugate
- Dividing complex numbers

3B. Geometry & Trigonometry

3B.1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons

- Squares, rectangles, parallelograms, the rhombus, trapezoids, & other quadrilaterals
- Computing area, perimeter, & lengths
- Polygons (regular and otherwise)

3B.2. Problem Solving Involving Triangles, Quadrilaterals and Polygons

- Using all geometry topics covered to date
- Specifically covers anything from 1B.1, 1B.2, 2B.1, 2B.2, and 3B.1

3B.3. Transformations in the Plane

- Scaling
- Reflections, typically across horizontal or vertical lines
- Rotations, typically by multiples of 90 degrees
- Relationship between line slope and the tan function

3B.4. Trigonometric Identities

- Angle sum and difference formulas
- Double angle identities
- Half angle identities
- Solving trigonometric equations

3C. Counting, Probability & Statistics, Number Theory

3C.1. Counting Combinations

- Correcting for overcounting
- Counting combinations: $\binom{n}{r}$

3C.2. More Probability

- Calculating probabilities using combinations & other counting techniques
- Conditional probability

3C.3. Pascal's Triangle and the Binomial Theorem

- Pascal's Triangle
- The Binomial Theorem: coefficients of $(x + y)^n$

3C.4. Calculating Digits, especially the Last Digit

- Finding the last digit of a sum, product, or power of integers
- Counting trailing zeros of products of integers

3A.1. The Quadratic Formula

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3B.2. Problem Solving involving Triangles, Quadrilaterals and Polygons

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- Counting trailing zeros of products of integers

Meet 4 Topics Breakdown

4A. Algebra

4A.1. Optimization Problems

- Minimum or maximum value of quadratic expressions (e.g. by completing the square)
- The Arithmetic-Geometric mean inequality

4A.2. Roots of Polynomial Equations

- Vieta's formula for the sum of roots of a polynomial
- Vieta's formula for the product of roots of a polynomial
- Vieta's formula for relationships between roots and other coefficients of a polynomial

4A.3. Geometric Sequences and Series

- Geometric sequence definition
- Finding the common ratio
- Finding the n th term
- Geometric series definition
- Calculating the sum of a geometric series
- Geometric sequence & series problem solving

4A.4. More Logarithms & Exponents

- The change of base formula: $\log_a b = \frac{\log_c b}{\log_c a}$
- Solving exponential equations
- Solving logarithmic equations

4B. Geometry & Trigonometry

4B.1. Area, Perimeter, and Angles in Circles

- Sectors
- Circular segments
- Central & inscribed angles
- The Inscribed Angle Theorem

4B.2. Lengths involving Circles

- Internal and External tangent lines
- Power of a Point
- Cyclic quadrilaterals
- Angle relationships in cyclic quadrilaterals (e.g. opposite angles)
- Problem solving in cyclic quadrilaterals (e.g. Ptolemy's Theorem, Brahmagupta's Formula)

4B.3. Analytic Geometry of Circles

- Circle equations
- Intersections between circles

4B.4. Law of Sines & Law of Cosines

- The Law of Sines
- The Law of Cosines
- Finding lengths and angles in triangles and other geometric figures using these laws

4C. Counting, Probability & Statistics, Number Theory

4C.1. More Counting & Probability

- Counting integer solutions to $x_1 + x_2 + \dots + x_n = k$ ("sticks and stones"), including positive and non-negative cases
- Counting more than two groups (multinomials)
- Counting paths in a grid
- Counting & probability problems using all topics to date

4C.2. Expected Value

- Definition based on individual probabilities
- Expected value problem solving

4C.3. Remainders & Modular Arithmetic

- Calculating remainders
- Modular arithmetic notation
- Remainders of sums, products, and powers of integers

4C.4. Finding Integer Solutions

- Finding integer solutions by factoring & casework
- Finding integer solutions by completing the rectangle ("Simon's Favorite Factoring Trick")

4A.1. Optimization Problems

- Minimum or maximum value of quadratic expressions (e.g. by completing the square)
- The Arithmetic-Geometric mean inequality

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- Vieta's formula for the product of roots of a polynomial
- Vieta's formula for relationships between roots and other coefficients of a polynomial

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- Finding the n th term
- Geometric series definition
- Calculating the sum of a geometric series
- Geometric sequence & series problem solving

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- The change of base formula: $\log_a b = \frac{\log_c b}{\log_c a}$
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- Solving logarithmic equations

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- Finding integer solutions by completing the rectangle (“Simon’s Favorite Factoring Trick”)

Meet 5 Topics Breakdown

5A. Algebra

5A.1. Non-Linear Equations and Systems of Equations

- Solving radical equations
- Identifying extraneous roots
- Solving more complex equations or systems of equations (combining any techniques used to date)

5A.2. Functional Equations

- Solving for a function based on given properties of the function
- Finding function value for a specific input given properties of the function
- Domain and Range; Function Composition; Inverse Functions
- Functional Operations (Invented operators)

5A.3. Sums of Powers of Integers

- Formula for $1 + 2 + \dots + n$
- Formula for $1^2 + 2^2 + \dots + n^2$
- Formula for $1^3 + 2^3 + \dots + n^3$
- Finding related sums using these formulae

5A.4. General Sequences & Series

- The Fibonacci sequence
- Recursively defined sequences
- Telescoping sums
- General sequence & series problem solving

5B. Geometry & Trigonometry

5B.1. 3-Dimensional Geometry

- Triangular and polygonal prisms & pyramids, cylinders, cones, spheres
- Calculating volume and surface area

5B.2. Geometry Problem Solving

- Problem solving using all Geometry topics
- Could include any topics from subtopics 1 and 2 of previous B events
- Specifically topics 1B.1, 1B.2, 2B.1, 2B.2, 3B.1, 3B.2, 4B.1, 4B.2

5B.3. Analytic Geometry of Conic Sections

- Axis of symmetry
- Focus-directrix definition of a parabola
- Parabola equations
- Focus-focus definition of an ellipse
- Ellipse equations
- Focus-focus definition of a hyperbola
- Hyperbola equations

5B.4. Geometry of Complex Numbers

- The Complex Plane
- Modulus of Complex Numbers
- Distance in the Complex Plane
- Polar representation of a complex number, relationship to trigonometry
- De Moivre's Theorem

5C. Counting, Probability & Statistics, Number Theory

5C.1. Divisor Arithmetic

- Counting divisors of a number
- Sum of divisors of a number

5C.2. Venn Diagrams & the Principle of Inclusion-Exclusion

- Counting with Venn diagrams (2 or 3 sets)
- Counting using the principle of inclusion-exclusion (any number of sets)

5C.3. Geometric Probability

- Calculating probability as a fractional area

5C.4. More Integer Solutions

- More general diophantine equations
- Solving modular equations and systems

5A.1. Non-Linear Equations and Systems of Equations

- Solving radical equations
- Identifying extraneous roots
- Solving more complex equations or systems of equations (combining any techniques used to date)

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- Solving for a function based on given properties of the function
- Finding function value for a specific input given properties of the function
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- Formula for $1^2 + 2^2 + \dots + n^2$
- Formula for $1^3 + 2^3 + \dots + n^3$
- Finding related sums using these formulae

5A.4. General Sequences & Series

- The Fibonacci sequence
- Recursively defined sequences
- Telescoping sums
- General sequence & series problem solving

5B.1. 3-Dimensional Geometry

- Triangular and polygonal prisms & pyramids, cylinders, cones, spheres
- Calculating volume and surface area

5B.2. Geometry Problem Solving

- Problem solving using all Geometry topics
- Could include any topics from subtopics 1 and 2 of previous B events
- Specifically topics 1B.1, 1B.2, 2B.1, 2B.2, 3B.1, 3B.2, 4B.1, 4B.2

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- Axis of symmetry
- Focus-directrix definition of a parabola
- Parabola equations
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- Counting using the principle of inclusion-exclusion (any number of sets)

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- Calculating probability as a fractional area

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- More general diophantine equations
- Solving modular equations and systems

Thank you!

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