Minnesota State High School Mathematics League

Newsletter

Issue #21 December 7, 2020

A message from the Executive Director, Tom Young

Two meets down, three to go! Meet Two certainly went better than Meet One. Special thanks to Gary Kannel for all of his work improving the online system between meets. And thanks to the coaches and students who adapted to different time slots to help ensure the server didn't crash. ***<u>Make sure you sign up for time slots for Meet Three.</u>***

And thanks to the problem writers who adapted and rewrote the problems. The integer answers have made correcting easier. The efforts Tom Kilkelly made to thoroughly vet the problems significantly reduced challenges. Many thanks to him and his team.

And ***thanks to the students*** for their efforts in solving great problems. We strive to make the first two problems accessible to students who have studied the topics. The second two are tougher but accessible to students with deep understanding and quick insights. Those two problems can be pretty tough and it astounds me how many students are able to solve them in the time given. The future mathematical leaders of the world are being shaped before our eyes! Look later in the newsletter for a new feature where a former Math Team standout reminisces about the impact of Math League.

Note the information concerning the State Tournament. *It will be online this year!* Enter the t-shirt design contest and the video contest for cash prizes!

I have said this before, but it bears repeating: I am impressed with the commitment to bringing extra-curricular math to Minnesota students.

Go Math Team!

Please send pictures of your team to be included in this space

A message from Tom Kilkelly, Head of the Problem Writing Team

Meet Two was a success! Most students wrote integer answers for their solutions. Students should note that they ***do not have to include units*** in their answers. All they have to do is enter the integer in the text box.

As a reminder, here are the conventions we are using this year:

Two expressions have been used extensively throughout this years' problem sets.

"… can be written as $a\sqrt{b}$ where b is square-free. Determine the value of a + b"

For b to be "square-free", it cannot have factors which are square numbers (other than 1). For example, as in the past, an answer of $\sqrt{12}$ would be unacceptable and students would have had to convert it to $2\sqrt{3}$ to receive credit. This year the student must still convert but the student must submit the answer 5 to receive credit. (N.B. If $\sqrt{b}$ cannot be simplified, the problem would state "… can be written as $\sqrt{b}$, where b is square-free." And the answer to submit would be b)

"… can be written as $\frac{p}{q}$, where p and q are relatively prime integers. Determine the value of p + q."

For example, as in the past, an answer of $\frac{6}{8}$ would be unacceptable and students would need to simplify it to $\frac{3}{4}$ to receive credit. This year the student must submit the answer 7 to receive credit.

There is an area of caution with this type of fraction formatting of which all students should be made fully aware:

<u>Negative Rational Answers</u>

Although we all know that $-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$ in order to create a unique answer, all students should be made aware that for this competition, the negative sign MUST be assigned to the numerator and NOT to the denominator.

So if the answer is $-\frac{3}{5}$ the student must submit the answer 2 to receive credit and if the answer is $-\frac{5}{3}$, the student must submit the answer – 2 to receive credit.

In Meet Two, there was a challenge as to whether a point on the positive y-axis is part of Quadrant I. By definition, Quadrant I contains points whose coordinates are both positive. Therefore, the challenge was denied.

Two other challenges to Team #2 were denied, but caused spirited reflection. Two issues were at the center of the challenge.

ONE can points "within" a triangle be "on" the triangle? and,
TWO can cevians be drawn along the sides of a triangle?

The students that challenged argued for the affirmative in both cases. Judges ruled in the negative on both.

One goal of Math League is to foster curiosity. Team problem #4 from Meet 2 could have been stated:

A triangle has vertices $A(a_x, a_y)$, $B(b_x, b_y)$, and $C(c_x, c_y)$. A circle, circumscribing this triangle, is centered at $P(m, n)$. Find (m,n) in terms of the coordinates of A, B, and C. This is a rather difficult algebraic exercise! Give it a go! The answer can be found at

https://en.wikipedia.org/wiki/Circumscribed_circle

Good luck to students on Meet Three. A prepared student will have studied double angle formulas and parallel and coincident lines!

MN State High School Math League

2021 State Tournament
T-shirt Design Contest

Prize: **$50 VISA Gift Card and a Free T-shirt**

How to enter:

Submit a one-color design for the t-shirt front.
The design should include the words:

MN State High School Math League
State Tournament
March 15, 2021

- Email your design by **Feb. 1st** to: mathleague@augsburg.edu
- Accepted file format: pdf only
- Include your name, grade and school in the email submission.
- Winner will be notified by Feb. 17th via email.

Email mathleague@augsburg.edu with questions

MN State High School Math League
Math Team Video Contest

1st place: $200 to school's math team
2nd place: $150 to school's math team
3rd place: $100 to school's math team

Video Guidelines:

Produce a 90 second video explaining why you like to be involved in the Math League. Videos might include: student interviews, teacher endorsements, sample problems, or video of practices/meets.

Video Entry Submission:

Videos are due to the Math League Office (mathleague@augsburg.edu) by *March 1st, 2021*.

- **Videos contest entries must be sent and approved by the school math team coach.**
- **Winning schools will be notified by March 6, 2021.**
- **Winning videos will be shown at the State Tournament on March 15, 2021, uploaded to the Math League Facebook page, and may be used for other promotional purposes.**

Questions? Email mathleague@augsburg.edu

The Impact of Math Team

The call went out this summer to Math League alumni to Share Your Story. Here is one alumnus who shared:

Kelly Stifter

2011 Graduate of Spring Lake Park HS

Undergraduate Degree: BS Physics, University of Minnesota
Low Background Counting Facility (LBCF) in the Soudan Mine
Science Undergraduate Laboratory Intern (SULI) at Fermilab
Intern at CERN
Graduate Degree: PhD Physics, Stanford
Working on LZ Dark Matter Experiment

Life goals:

Looking to the future, Kelly's plan is to keep doing research. Her end goal is to become a faculty member at a large land grant institution so she can continue doing cutting edge research, and also get the opportunity to teach students in the classroom and mentor them in the lab.

Interesting avocations:

Outside of research, Kelly is deeply involved in science outreach to the public, public policy and advocacy efforts, and Equity & Inclusion efforts. She is also an avid hiker/backpacker, rock climber, and video game player (placing 3rd in the nation in a particular video game competition back in 2018).

The impact of Math Team on my life and learning:

I think Math Team informed me of what I wanted to do and what I *could do* with my future in several ways. First and foremost was that, even though I often wasn't an expert on the particular topics covered in the various meets throughout the year, I could study the topics ahead of time and still do well. This helped teach me that I didn't have to be a genius to succeed, I just had to be ambitious and work hard.

At the time, I didn't realize that the problems we were given in Math Team weren't like the actual study of Mathematics (at least what Math professors do on a daily basis). In fact, I think that what I do on a daily basis is much closer! Math Team encouraged my love of solving puzzles, which I now have to do all the time. It helped teach me to take the skills that I had learned and apply them in new ways to new types of problems.

Finally, it also taught me that I didn't want to work in an isolating environment. My favorite part of the Math Team experience was not taking the test, but spending time working on problems with my friends - learning from them, and also helping to teach them. This helped me realize I wanted my future work to be highly collaborative.

Did you know that our Math League logo illustrates Morley's theorem?

cited from: https://www.cut-the-knot.org/triangle/Morley/Webster.shtml#:~:text=Morley's%20theorem%20states%20that%20%CE%94A,sin(C%2F3)

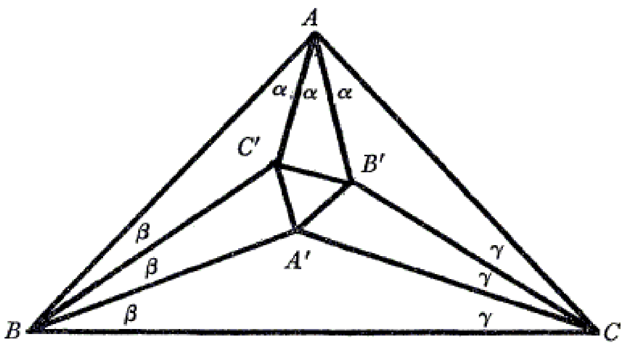
Morley's Theorem

The three points of intersection of the adjacent trisectors of the angles of any triangle form an equilateral triangle.

Proof

This proof appeared in *Mathematics Magazine*, Vol. 43, No. 4 (Sep., 1970), pp. 209-210.

Let ABC be a triangle with inradius r and circumradius R, and let the adjacent trisectors of angles A, B, C meet in A', B', C' as illustrated below. Morley's theorem states that ΔA'B'C' is equilateral. The proofs of this theorem, which are usually given, do not include calculation of the side of this triangle.



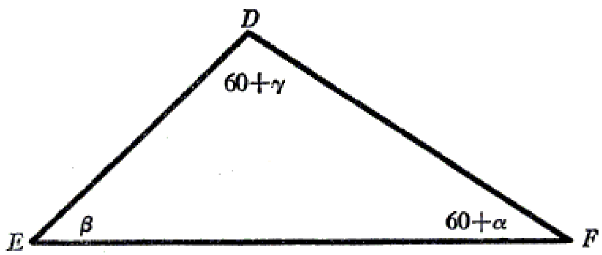
In this note we prove Morley's theorem by showing that the side of this triangle is 8R sin(A/3)sin(B/3)sin(C/3). This is a particularly interesting result, when one recalls *the formula* r = 4R sin(A/2)sin(B/2)sin(C/2).

In any triangle ABC, we have a = 2R sin(A), etc. Let A = 3α, B = 3β, C = 3γ. Then applying the sine rule to A'BC we obtain

$$A'B = 2R \sin(3\alpha) \sin\gamma / \sin(\beta + \gamma)$$

$$= 2R (4\sin\alpha \sin(60° + \alpha) \sin(60° - \alpha) \sin\gamma) / \sin(60° - \alpha)$$

$$= 8R \sin\alpha \sin\gamma \sin(60° + \alpha).$$



Similarly, BC' = 8R sinα sinγ sin(60° + γ). Consider now triangle DEF shown above, where DE = 8R sinα sinγ sin(60° + α). It follows, using the *sine rule* on DEF, that EF = 8R sinα sinγ sin(60° + γ) and DF = 8R sinα sinβ sinγ. However, triangles A'BC' and DEF are congruent by *SAS*, so A'C' = DF = 8R sinα sinβ sinγ Thus, by the symmetry of the expression, the triangle A'B'C' is equilateral and Morley's theorem is proved.

Summer Coaches Conference 2021
Date: TBD

Last summer, we had to postpone our 40-year celebration due to the pandemic. Hopefully we will be able to hold a celebration next summer honoring our new Hall of Famers and toasting to another 40 years!

2021 Summer Math Institute

Dates TBD at Augsburg University

The League hopes to offer two one-week programs of the Summer Mathematics Institute in 2021. The pandemic will shape our decision; we think we can offer the program, but perhaps not a residential one.

One would be for students entering grades 7-9 in fall of 2021. **The topic would be Knots! and taught by Annie Perkins.**
The other would be for students entering grades 10-12 in fall of 2021. **The topic would be Number Theory in Math League and the AMC and taught by Ken Suman.**

Stay tuned!!

The Roberts Award Scholarship

The Roberts Award Scholarship(s) were established in honor of the League founder, Dr. Wayne Roberts of Macalester College.

The Scholarship(s) are offered to help offset the costs for students interested in attending an out-of-state math opportunity. They are offered once each year. A set amount of funds will be available each year, and multiple awards are possible.

Deadline to apply for this season is April 30, 2021

Applications can be found on our web site at: http://mnmathleague.org/?page_id=1033

2020 – 2021 State Tournament will be Online

The executive committee voted Sunday, December 6th, to approve a motion that the State Tournament on March 15, 2021 will be held online. The Invitational, Math Bowl, and Team portions of the Tournament will all be held via a Zoom-like platform in conjunction with our online delivery system. Some details still need to be worked out, but teams that qualify should be ready for a great day of mathematics!

Common Meet Protocols

Coaches must verify each student's score, and mark team done with each event.

Coaches can give credit if:

- the student includes units in the answer. (e.g. 6 degrees when the answer should be 6)
- there is an issue like adding a space to the answer (SPACE 6 instead of 6).
- the student writes something akin to x = 6.

All other discrepancies should be challenged. For instance, coaches should **not** give credit to mistyped answers even if the students have the correct answer on their scratch work. Challenges regarding incorrectly typed answers were denied unless there were issues with the computer system not working.

Students should be reminded that all answers are integers.

Also, the students should be told how the computer system registers their answers. The textbox for submitting the answer is blank when the event starts. When students enter an answer, the textbox turns yellow. (NOTE: this is a change in the color) When they click away from that textbox, it will turn white and the answer should stay displayed. This indicates that the system has registered their answer. When students finish the event, only then should they hit submit. If they hit submit before they are done, they are locked out. When the 15-minute time limit expires, answers are automatically submitted. Students do not have to hit submit if they are timed out. Students that ask to enter an answer after the time limit expires, claiming that they didn't get a chance to enter their answer, should not be allowed to challenge that.

Students should be reminded that calculators are not allowed on individual events.

When auditing student responses, we noted instances of answers like 3.6 E–15. That is worrisome. One coach remarked that we are actually only "wink, wink" enforcing that rule. That is **not** our position. We see this as an opportunity to show students that ethical behavior is valued. It is up to each coach to monitor their students and help them see the value in maintaining the integrity of the process.

Even if coaches verify results, mistakes will be made.

We are able to see all the answers submitted for a particular problem. Coaches missed correct answers and didn't give the student credit and, on the flip side, gave credit when it shouldn't have been given. We sent emails to those coaches noting the discrepancies. We will do this for each meet to make the scores are as accurate as possible.

Certain online calculators are allowed on the team event.

Some students argued that since they are in distance learning mode, they cannot access their school's calculators and therefore should be allowed to access online calculators. We feel that students can use the calculators at http://minnesota.pearsonaccessnext.com/stand-alone-calculators/ during the team event.

Zoom-like tools are allowed on the team event.

Teams can use the share screen, or other Zoom-like tools when they are working as a team. However, if the meeting platform contains a calculator, it cannot be accessed.

Problem Corner

an effort to spur conversation

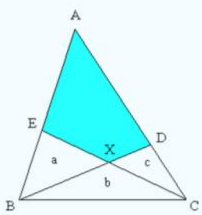
If you'd like to contribute a problem or send in a solution, email tomyoungmathman@gmail.com

Student solutions encouraged!

Newsletter 20 Puzzler

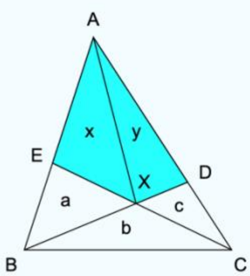
2. Triangular area ☆☆

In △ ABC, produce a line from B to AC, meeting at D, and from C to AB, meeting at E. Let BD and CE meet at X.
Let △ BXE have area a, △ BXC have area b, and △ CXD have area c. Find the area of quadrilateral AEXD in terms of a, b, and c.



Solution:

Solution to puzzle 2: Triangular area



△BXE has area a, △BXC has area b, and △CXD has area c.

We will use the fact that the area of a triangle is equal to ½ × *base* × *perpendicular height*. Any side can serve as the base, and then the perpendicular height extends from the vertex opposite the base to meet the base (or an extension of it) at right angles.

Consider BXE and BXC, with collinear bases EX and XC, respectively.
The triangles have common height; therefore EX/XC = a/b.
Similarly, considering BXC and CXD, with respective bases BX and XD, BX/XD = b/c.

Now draw line AX. Let △AXE have area x and △AXD have area y.

Consider AXB and AXD, with bases BX and XD, such that BX/XD = b/c.
Since AXB and AXD have common height, we have (a + x)/y = b/c.
Similarly, considering AXE and AXC, with collinear bases EX and XC, x/(y + c) = a/b.

Hence, by = cx + ac and bx = ay + ac.

Solving these simultaneous equations, we obtain

$x = ac(a + b)/(b^2 - ac)$, $y = ac(b + c)/(b^2 - ac)$.

Therefore the area of quadrilateral AEXD = x + y = $\dfrac{ac(a + 2b + c)}{b^2 - ac}$

NEWSLETTER # 21 PUZZLER: (from http://www.qbyte.org/puzzles/puzzle02.html)

15. Infinite product ☆☆

Find the value of the infinite product

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \ldots \times \frac{k^3 - 1}{k^3 + 1} \times \ldots$$