



A message from the Executive Director, Tom Young

Three meets down, two to go! And then the State Tournament! Some minor glitches to deal with in Meet Three but, overall, the best yet. Division coordinators, we'd like to hear feedback from you regarding the contest and the problems. Use the Meet Op> Meet Feedback and help us improve!

Meet Four is January 25th. Sign up for time slots will start January 11th. I encourage teams to stay in their slot. We've got a good thing going that we don't want to mess up.

Note the information concerning the State Tournament. *It will be online this year!* Plans are being made to hold the State Tournament via Zoom or like product. If you think your team will qualify for the state tournament, and Zoom wouldn't work for your school, send me an email at tomyoungmathman@gmail.com.

Enter the t-shirt design contest and the video contest for cash prizes!

Check out the second entry for our continuing series highlighting previous Math Leaguers. If you know of a former student that would like to share their experiences, pass their name along.

Go Math Team!

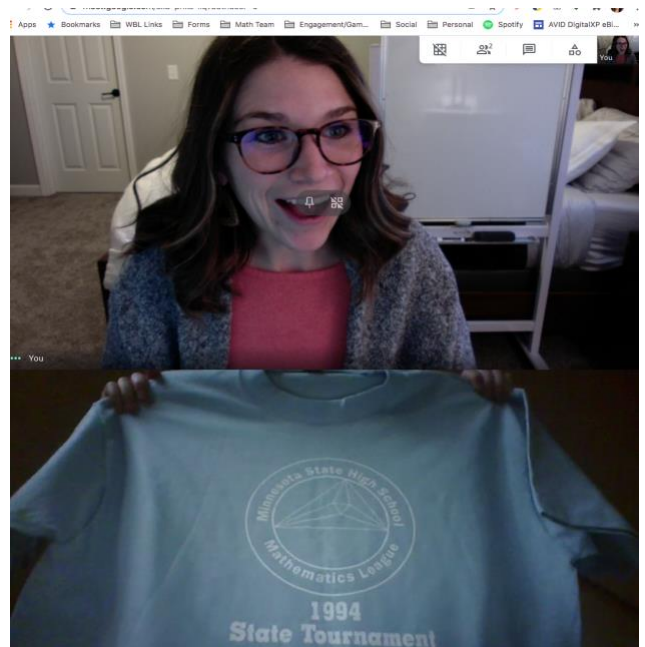
Speaking of former students, this picture was forwarded by teacher Kendra Herder from Central Middle School in White Bear Lake. Kendra wrote:

I thought you might enjoy a screenshot of a student sharing with me (after the meet today) her father's shirt from the MN State High School Math League. She said being a part of the Math Team is in her DNA. :)

And that student is Noelle Wilding (8th grader) and her sister is Sophia Wilding (6th grader) and their father is Matthew Wilding.

Thanks, Noelle, for sharing! Noelle and Sophia compete in the Junior High Math League.

We look forward to seeing them in the Math League as they follow their father's footsteps!



A message from Tom Kilkelly, Head of the Problem Writing Team

Meet Three was another success! Most students wrote integer answers for their solutions. Students should note that they *do not have to include units* in their answers. All they have to do is enter the integer in the text box.

As a reminder, here are the conventions we are using this year:

Two expressions have been used extensively throughout this years' problem sets.

"... can be written as $a\sqrt{b}$ where b is square-free. Determine the value of $a + b$ "

For b to be "square-free", it cannot have factors which are square numbers (other than 1). For example, as in the past, an answer of $\sqrt{12}$ would be unacceptable and students would have had to convert it to $2\sqrt{3}$ to receive credit. This year the student must still convert but the student must submit the answer 5 to receive credit. (N.B. If \sqrt{b} cannot be simplified, the problem would state "... can be written as \sqrt{b} , where b is square-free." And the answer to submit would be b)

"... can be written as $\frac{p}{q}$, where p and q are relatively prime integers. Determine the value of $p + q$."

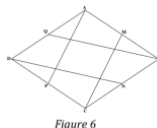
For example, as in the past, an answer of $\frac{6}{8}$ would be unacceptable and students would need to simplify it to $\frac{3}{4}$ to receive credit. This year the student must submit the answer 7 to receive credit.

There is an area of caution with this type of fraction formatting of which all students should be made fully aware:

Negative Rational Answers

*Although we all know that $-\frac{p}{q} = \frac{-p}{q} = \frac{p}{-q}$ in order to create a unique answer, all students should be made aware that for this competition, the negative sign **MUST** be assigned to the numerator and **NOT** to the denominator.*

So if the answer is $-\frac{3}{5}$ the student must submit the answer 2 to receive credit and if the answer is $-\frac{5}{3}$, the student must submit the answer - 2 to receive credit.



In Meet Three, Team #6 referred to this rhombus

where the obtuse angle = 120° .

A quick search through the Archives at scoringmnmathleague.org finds a similar problem in 2000-2001 Meet 3 Event B Problem 3 where the angle = 90° , i.e., the larger quadrilateral is a square. In both cases, the inner quadrilateral is one-fifth the area of the larger quadrilateral!

This brings up two ideas for the intrepid math-leaguers:

One: sometimes problems can be solved WLOG (without loss of generality) by adopting a special case of the initial conditions. If the square from 2000-2001 were "squeezed", it could be deformed to the 2020-2021 rhombus yet the area relationships should remain. There are Math League problems that can be solved by considering a special case and then reasoning that the special case holds for the conditions of the general case.

Two: A good mathematician gets intrigued by problems such as these. A good mathematician delves deeper. Is it always true that the inner quadrilateral is one-fifth the area of the larger?

The mathematician Rick Mabry investigated this question and his article titled *Crosscut Convex Quadrilaterals* can be found in *Mathematics Magazine*, Vol. 84, No. 1 (February 2011), pp. 16-25. In the article he proves that if the inner quadrilateral has at least 2 parallel sides, the area is always one-fifth the outer quadrilateral! He also proves, in general, the inner quadrilateral is at least one-sixth and at most one-fifth the area of the larger quadrilateral! Amazing!

Good luck to students on Meet Four. A prepared student will have studied secants of a circle, conic sections, and binomial expansion.

ALL STATE MATH TEAM UPDATE

As we all know, 2020 was a trying year. Perhaps fittingly, we are all trying our best. This certainly goes for our 2020-21 Minnesota All-State Math Team. Tri-captains Jason Wang of the Blake School, Alexis Yi of Edina High School, and David Zhang of Sartell High School recently led three consecutive Saturday-night (!) practices as 45 mathletes prepared for the Harvard-MIT Mathematics Online (HMMO) contest held 14-21 November 2020.

Top HMMO individual scores were from Matthew Chen of Wayzata High School and Linden Li of Mounds View High School (7/10 in General Round), and Sasha Hydrie of St. Paul Central High School and Linden Li (7/10 in Theme Round). Top sub-team scores were from the Minnesota A (Maroon) team of Kenneth Chen of East Ridge High School, Matthew Chen, Evan Erickson of the Erickson Homeschool Academy, Linden Li, and Kevin Yang of Wayzata (400/400 in Team Round, 167/399 in Guts Round).

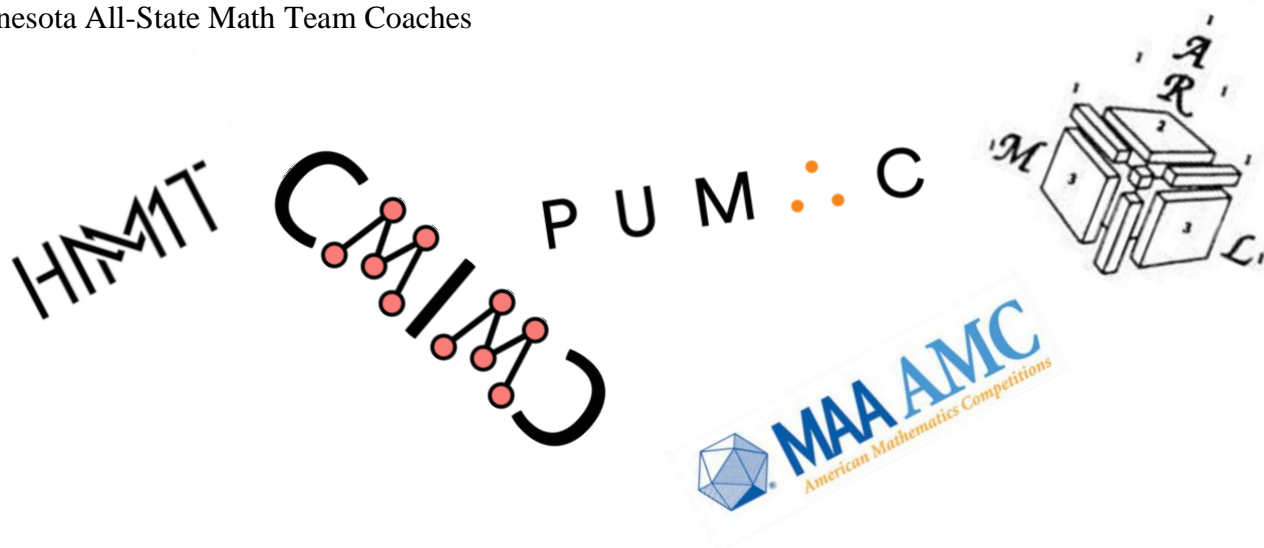
Upcoming team events, each of which will be held online, include the new Carnegie Mellon Informatics and Mathematics Competition (CMIMC) Programming contest on 30 January, the Harvard-MIT Mathematics Tournament (HMMT) in February, the traditional CMIMC mathematics contest in spring, and the Princeton University Mathematics Competition (PUMaC) in spring.

Long a highlight of the competition season, the American Regions Mathematics League (ARML) event, traditionally held in late May or early June, looks to resume in 2021. Details will be communicated when available.

Minnesota All-State Math Team selection is based upon several criteria, including Math League regular-season scores, Math League Invitational Event (at the State Tournament) scores, and AMC 10 or AMC 12 scores. Now would certainly be a good time to arrange your plans for taking the AMC 10 or AMC 12, scheduled for Thursday, 4 February (AMC 10A/12A) and Wednesday, 10 February (AMC 10B/12B). The 2021-22 team selection process will commence following the State Tournament.

Stephen MacLennan and Matthew Eggert

Minnesota All-State Math Team Coaches



The Impact of Math Team

The call went out this summer to Math League alumni to Share Your Story. Here is one alumnus who shared:

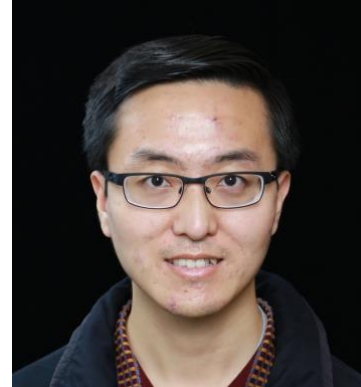
Shui Hu

2008 Graduate of Irondale HS

Undergraduate Degree: BS in computer science at Stanford

Graduate Degree: MS in computer science at Stanford

Currently works at : Amazon as an engineering manager.
Shui was also an engineering manager at Uber.



The impact of Math Team on my life and learning:

The Minnesota State High School Math League was a fixture of my middle and high school years. Somewhat unusually, I started participating in the High School Math League when I was in seventh grade around November 2002 and continued through my senior year of high school in 2008. During those six years (five complete seasons and one meet from the 2002-2003 season), I represented Irondale and Mounds View high schools. Participating in Math League was a valuable experience which honed my math skills, contributed to my choice to work in AI, helped me grow as a teenager, and was a source of many happy memories and friendships.

Math competition was the major focus of my life during the decade of the 2000s, and Math League was the most enjoyable of the competitions in which I participated. My favorite topics from Math League were geometry (Euclidean geometry was my favorite subject in competition math in general) and conic sections. I think Math League problems are very good for stimulating interest in learning more mathematics among a big portion of the high school student population.

Aside from the math problems themselves, I also found the people side of Math League very rewarding. Participating in the regular season, tournament, Math Bowl, and the subsequent ARML training sessions/event was genuinely fun. Most of my friendships from that period of my life were related to Math League and those are among my longest-lasting personal relationships today. I also learned a lot from the ARML coaches Tom Kilkelly, Bill Boulger, and Mike Reiners who not only volunteered many hours to train the state teams, but also made the whole process relaxed and very collaborative. Dr. Wayne Roberts, who founded the league, used to say in his address at every tournament that we should celebrate all of the participants, not just the winners of the competition. I have found that to be very far-sighted, because the greatest contribution of Math League, in my opinion, has been in fostering a community of people with love for technical subjects that utilize mathematics.

After high school, I always had a strong desire to use mathematics to solve problems. While attending college at Stanford, I got interested in how cognition worked. Eventually, this led me to machine learning, a field which relies heavily on higher mathematics. Over the last couple of years, I have been leading teams working on Natural Language Processing and applying that to conversational assistants. I chose this area because I think NLP has a lot of untapped potential that can be realized using advanced mathematics.

Summer Coaches Conference 2021

Date: TBD

Last summer, we had to postpone our 40-year celebration due to the pandemic. Hopefully we will be able to hold a celebration this summer honoring our new Hall of Famers and toasting to another 40 years!

2021 Summer Math Institute

Dates TBD at Augsburg University

The League hopes to offer two one-week programs of the Summer Mathematics Institute in 2021. The pandemic will shape our decision; we think we can offer the program, but perhaps not a residential one.

One would be for students entering grades 7-9 in fall of 2021. **The topic would be Knots! and taught by Annie Perkins.**

The other would be for students entering grades 10-12 in fall of 2021. **The topic would be Number Theory in Math League and the AMC and taught by Ken Suman.**

Stay tuned!!

The Roberts Award Scholarship

The Roberts Award Scholarship(s) were established in honor of the League founder, Dr. Wayne Roberts of Macalester College.

The Scholarship(s) are offered to help offset the costs for students interested in attending an out-of-state math opportunity. They are offered once each year. A set amount of funds will be available each year, and multiple awards are possible.

Deadline to apply for this season is April 30, 2021

Applications can be found on our web site at: http://mnmathleague.org/?page_id=1033

2020 – 2021 State Tournament will be Online

The executive committee voted Sunday, December 6th, to approve a motion that the State Tournament on March 15, 2021 will be held online. The Invitational, Math Bowl, and Team portions of the Tournament will all be held via a Zoom-like platform in conjunction with our online delivery system. Some details still need to be worked out, but teams that qualify should be ready for a great day of mathematics!



MN State High School Math League

2021 State Tournament T-shirt Design Contest

Prize: **\$50 VISA Gift Card and a Free T-shirt**

How to enter:

Submit a one-color design for the t-shirt front.

The design should include the words:

MN State High School Math League
State Tournament
March 15, 2021

- Email your design by **Feb. 1st** to: mathleague@augsborg.edu
- Accepted file format: pdf only
- Include your name, grade and school in the email submission.
- Winner will be notified by Feb. 17th via email.

Email mathleague@augsborg.edu with questions



MN State High School Math League Math Team Video Contest

1st place: \$200 to school's math team

2nd place: \$150 to school's math team

3rd place: \$100 to school's math team

Video Guidelines:

Produce a 90 second video explaining why you like to be involved in the Math League. Videos might include: student interviews, teacher endorsements, sample problems, or video of practices/meets.

Video Entry Submission:

**Videos are due to the Math League Office
(mathleague@augsborg.edu)
by *March 1st, 2021.***

- **Videos contest entries must be sent and approved by the school math team coach.**
- **Winning schools will be notified by March 6, 2021.**
- **Winning videos will be shown at the State Tournament on March 15, 2021, uploaded to the Math League Facebook page, and may be used for other promotional purposes.**

Questions? Email mathleague@augsborg.edu

Great Idea from a teacher coach

In case the internet fails during a meet, one of our coaches, Cassie Erdmann, has her students fill out a form (pictured) that is used as a backup. Also, she has students initial a statement that affirms they didn't cheat. Great ideas, Cassie!

Use this document to track the final answers you submit to the online meet portal. No work is required to be shown. This is only for backup purposes.

Event A	Event B
#1: <u>66</u>	#1: <u>12</u>
#2: <u>64</u>	#2: <u>460</u>
#3: <u>30</u>	#3: <u>4</u>
#4: <u>37</u>	#4: <u>15</u>

Common Meet Protocols

Coaches must verify each student's score, and mark team done with each event.

Coaches can give credit if:

- the student includes units in the answer. (e.g. 6 degrees when the answer should be 6)
- there is an issue like adding a space to the answer (SPACE 6 instead of 6).
- the student writes something akin to $x = 6$.

All other discrepancies should be challenged. For instance, coaches should **not** give credit to mistyped answers even if the students have the correct answer on their scratch work. Challenges regarding incorrectly typed answers were denied unless there were issues with the computer system not working.

Students should be reminded that all answers are integers.

Also, the students should be told how the computer system registers their answers. The textbox for submitting the answer is blank when the event starts. When students enter an answer, the textbox turns yellow. (NOTE: this is a change in the color) When they click away from that textbox, it will turn white and the answer should stay displayed. This indicates that the system has registered their answer. When students finish the event, only then should they hit submit. If they hit submit before they are done, they are locked out. When the 15-minute time limit expires, answers are automatically submitted. Students do not have to hit submit if they are timed out. Students that ask to enter an answer after the time limit expires, claiming that they didn't get a chance to enter their answer, should not be allowed to challenge that.

Students should be reminded that calculators are not allowed on individual events.

When auditing student responses, we noted instances of answers like $3.6 \text{ E}-15$. That is worrisome. One coach remarked that we are actually only "wink, wink" enforcing that rule. That is **not** our position. We see this as an opportunity to show students that ethical behavior is valued. It is up to each coach to monitor their students and help them see the value in maintaining the integrity of the process.

Even if coaches verify results, mistakes will be made.

We are able to see all the answers submitted for a particular problem. Coaches missed correct answers and didn't give the student credit and, on the flip side, gave credit when it shouldn't have been given. We sent emails to those coaches noting the discrepancies. We will do this for each meet to make the scores as accurate as possible.

Certain online calculators are allowed on the team event.

Some students argued that since they are in distance learning mode, they cannot access their school's calculators and therefore should be allowed to access online calculators. We feel that students can use the calculators at <http://minnesota.pearsonaccessnext.com/stand-alone-calculators/> during the team event.

Zoom-like tools are allowed on the team event.

Teams can use the share screen, or other Zoom-like tools when they are working as a team. However, if the meeting platform contains a calculator, it cannot be accessed.

Problem Corner

an effort to spur conversation

If you'd like to contribute a problem or send in a solution, email tomyoungmathman@gmail.com

Student solutions encouraged!

Newsletter 21 Puzzler:

(from <http://www.qbyte.org/puzzles/puzzle02.html>)

15. Infinite product ★★

Find the value of the infinite product

$$P = \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \dots \times \frac{k^3 - 1}{k^3 + 1} \times \dots$$

Solution, taken from <http://www.qbyte.org/puzzles/p015s.html>:

Factorizing numerator and denominator, we have

$$k^3 - 1 = (k - 1)(k^2 + k + 1)$$

$$k^3 + 1 = (k + 1)(k^2 - k + 1)$$

Note that $k^2 - k + 1 = (k - 1)^2 + (k - 1) + 1$, and so $k^3 + 1 = [(k - 1) + 2][(k - 1)^2 + (k - 1) + 1]$, allowing cancellation of the quadratic factor across successive terms, and of the linear factor across "next but one" terms.

We can now calculate P_n , the partial product of the first $n - 1$ terms.

$$\begin{aligned} P_n &= \frac{7}{9} \times \frac{26}{28} \times \frac{63}{65} \times \dots \times \frac{n^3 - 1}{n^3 + 1} \\ &= \left(\frac{1}{3} \times \frac{7}{3}\right) \times \left(\frac{2}{4} \times \frac{13}{7}\right) \times \left(\frac{3}{5} \times \frac{21}{13}\right) \times \dots \times \left(\frac{n-1}{n+1} \times \frac{n^2 + n + 1}{n^2 - n + 1}\right) \\ &= \frac{2}{3} \times \frac{n^2 + n + 1}{n(n+1)} \\ &= \frac{2}{3} \times \left(1 + \frac{1}{n(n+1)}\right) \end{aligned}$$

As $n \rightarrow \infty$, $P_n \rightarrow 2/3$.

That is, the infinite product, P , converges to $2/3$; $P = P_\infty = 2/3$.

New Puzzler

What is the longest sequence of *distinct* numbers between 1 and 99 [inclusive] you can find such that if X immediately follows Y then either X divides Y or Y divides X ?

For instance: 1, 5, 10, 20, 40, 8, 16, 4, 12, 24, 3, 9, 18 is such a sequence with 13 terms.

submitted to seqfan list-serv 12/26/2020 by Peter Luschny from "Xmas-challenge from Thien An."