

2024-25 MSHSML Topics

A - Algebra	B - Geometry & Trigonometry	C - Counting, Probability & Statistics, Number Theory
1A 1. Decimals, Fractions, and Percents 2. One Variable Linear Equations & Inequalities 3. Exponent Rules 4. Square Roots and Radicals	1B 1. Angles & Angle Relationships 2. Triangle Similarity and Congruence 3. Analytic Geometry of a Straight Line 4. Trigonometry Basics	1C 1. Basic Counting 2. Statistical Measures 3. Prime Factorization & Divisibility Rules 4. GCD and LCM
2A 1. Systems of Linear Equations 2. Binomials and Quadratics 3. Absolute Value 4. The Logarithm	2B 1. Area, Perimeter, and Lengths in Triangles 2. Right Triangles 3. Analytic Geometry of Points and Lines 4. More Elementary Trigonometry	2C 1. Counting Permutations and Independent Events 2. Analyzing Data 3. Basic Probability 4. Base n Arithmetic
3A 1. The Quadratic Formula 2. Polynomials 3. Arithmetic Sequences & Series 4. Complex Number Arithmetic	3B 1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons 2. Problem Solving involving Triangles, Quadrilaterals and Polygons 3. Transformations in the Plane 4. Trigonometric Identities	3C 1. Counting Combinations 2. More Probability 3. Pascal's Triangle and the Binomial Theorem 4. Calculating Digits, especially the Last Digit
4A 1. Optimization Problems 2. Roots of Polynomial Equations 3. Geometric Sequences & Series 4. More Logarithms & Exponents	4B 1. Area, Perimeter, Angles in Circles 2. Lengths involving Circles 3. Analytic Geometry of Circles 4. Law of Sines & Law of Cosines	4C 1. More Counting & Probability 2. Expected Value 3. Remainders & Modular Arithmetic 4. Finding Integer Solutions
5A 1. Non-Linear Equations and Systems of Equations 2. Functional Equations 3. Sums of Powers of Integers 4. General Sequences & Series	5B 1. 3-Dimensional Geometry 2. Geometry Problem Solving 3. Analytic Geometry of Conic Sections 4. Geometry of Complex Numbers	5C 1. Divisor Arithmetic 2. Venn Diagrams & the Principle of Inclusion-Exclusion 3. Geometric Probability 4. More Integer Solutions

Notes:

- Problems may draw on topics from any previous meet.
- Logic or puzzle type problems may appear in any event throughout the season.

Meet 1 Topics Breakdown

1A. Algebra

1A.1. Decimals, Fractions, and Percents

- Adding, subtracting, multiplying, or dividing fractions and decimals
- Reducing fractions to lowest terms
- Converting fractions to decimals
- Converting decimals (terminating or repeating) to fractions
- Percent increase & decrease
- Ratios & Proportions
- Interest problems

1A.2. One Variable Linear Equations & Inequalities

- Solving linear equations in one variable
- One variable linear inequalities
- Word problems leading to linear equations or inequalities
- Rate problems (distance = rate \times time)
- Unit Conversion

1A.3. Exponent Rules

- Positive integer exponents
- $a^m a^n = a^{m+n}, \frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^{-n} = \frac{1}{a^n}, a^0 = 1$

1A.4. Square Roots & Radicals

- Square roots
- Cube roots & higher roots
- Simplifying square roots & radicals of integers
- Adding, subtracting, multiplying, and dividing radicals
- Rationalizing simple radical denominators
- Radicals as exponents: $\sqrt[n]{a} = a^{\frac{1}{n}}$

1B. Geometry & Trigonometry

1B.1. Angles & Angle Relationships

- Angle sums in triangles and polygons
- Parallel lines and angle relationships

1B.2. Triangle Similarity and Congruence

- Conditions for congruence: SSS, SAS, ASA, AAS
- Conditions for similarity: AA, SAS, SSS
- Problem solving in triangles using Similarity and Congruence

1B.3. Analytic Geometry of a Straight Line

- Slope
- Slope-intercept form of a straight line
- Point-slope form of a straight line
- Midpoint of a segment

1B.4. Trigonometry Basics

- Degrees, radians, and converting between the two
- Right Triangle trig definitions
- Unit Circle trig definitions

1C. Counting, Prob. & Stats, Num. Theory

1C.1. Basic Counting

- Counting lists
- Counting by cases
- Complementary counting

1C.2. Statistical Measures

- Average/mean
- Median
- Mode

1C.3. Prime Factorization & Divisibility Rules

- Testing for divisibility by 2 through 12 (except 7)
- Prime factorization (using divisibility rules and testing primes up to square root)

1C.4. GCD and LCM

- Calculating the Greatest Common Divisor by factoring
- Calculating the Least Common Multiple by factoring
- Calculating Greatest Common Divisor using the Euclidean Algorithm
- $\text{gcd}(a, b) \times \text{lcm}(a, b) = ab$

Meet 2 Topics Breakdown

2A. Algebra

2A.1. Systems of Linear Equations

- Two (or occasionally more) variable linear systems of equations
- Word problems leading to systems of equations

2A.2. Binomials and Quadratics

- Multiplying binomials
- Rationalizing denominators using conjugates
- Difference of squares factorization
- Factoring a quadratic as a product of binomials
- Solving quadratic equations by factoring

2A.3. Absolute Value

- Solving absolute value equations and inequalities in one variable
- Representation on the Number Line

2A.4. The Logarithm

- Definition of logarithm
- Relationship to exponents
- $\log_n(ab) = \log_n a + \log_n b$
- $\log_n\left(\frac{a}{b}\right) = \log_n a - \log_n b$
- $\log_n(a^k) = k \log_n a$

2B. Geometry & Trigonometry

2B.1. Area, Perimeter, and Lengths in triangles

- Base-height area formula
- Medians, angle bisectors, altitudes
- Triangle Inequality
- Heron's Formula for triangle area
- Triangle theorems: Angle Bisector, Stewart, Ceva, Menelaus

2B.2. Right Triangles

- The Pythagorean theorem
- 30-60-90 and 45-45-90 triangles
- Common Pythagorean Triples

2B.3. Analytic Geometry of Points and Lines

- Distance between points
- Finding intersection points of lines
- Systems of inequalities used to define a region in the plane
- Areas of polygons on a grid
- Distance from a point to a line

2B.4. More Elementary Trigonometry

- Trig functions of common angles
- Basic identities: $\sin^2 \theta + \cos^2 \theta = 1$,
 $\tan \theta = \frac{\sin \theta}{\cos \theta}$, $\csc \theta = \frac{1}{\sin \theta}$,
 $\sin \theta = \cos\left(\frac{\pi}{2} - \theta\right)$, etc.

- Graphs of trig functions (sin, cos, tan)
- Inverse trigonometric functions
- Triangle area using trigonometry:

$$A = \frac{1}{2}ab \sin C$$

- Solving trigonometric equations

2C. Counting, Probability & Statistics, Number Theory

2C.1. Counting Permutations and Independent Events

- The multiplication principle for counting
- Counting Permutations: ${}_n P_r$
- Factorials
- Knowing when to add and when to multiply

2C.2. Analyzing Data

- Analyzing data in tables
- Analyzing data in charts
- Analyzing data in graphs

2C.3. Basic Probability

- Definition of probability
- Calculating probabilities using basic counting & permutations

2C.4. Base n Arithmetic

- Base n numbers and base n arithmetic
- Converting base n numbers to and from base 10
- Converting base n numbers to and from other bases

Meet 3 Topics Breakdown

3A. Algebra

3A.1. The Quadratic Formula

- Solving quadratics by completing the square
- Using the Quadratic Formula to solve quadratic equations
- The discriminant and character of roots
- Quadratic Inequalities

3A.2. Polynomials

- Multiplying polynomials
- Finding integer or rational roots of polynomials (the Rational Root Theorem)
- Factoring polynomials based on a known root; The Remainder Theorem
- Sum and difference of cubes factorization
- Sum and difference of odd powers factorization
- Solving polynomial equations
- Simplifying rational expressions (including Polynomial Division)
- Solving rational equations

3A.3. Arithmetic Sequences and Series

- Arithmetic sequence definition
- Finding the common difference
- Finding the n th term
- Arithmetic series definition
- Calculating the sum of an arithmetic series
- Arithmetic sequence & series problem solving

3A.4. Complex Number Arithmetic

- Adding and Subtracting complex numbers
- Multiplying complex numbers
- The Complex Conjugate
- Dividing complex numbers

3B. Geometry & Trigonometry

3B.1. Area, Perimeter, and Lengths in Quadrilaterals & Polygons

- Squares, rectangles, parallelograms, the rhombus, trapezoids, & other quadrilaterals
- Computing area, perimeter, & lengths
- Polygons (regular and otherwise)

3B.2. Problem Solving Involving Triangles, Quadrilaterals and Polygons

- Using all geometry topics covered to date
- Specifically covers anything from 1B.1, 1B.2, 2B.1, 2B.2, and 3B.1

3B.3. Transformations in the Plane

- Scaling
- Reflections, typically across horizontal or vertical lines
- Rotations, typically by multiples of 90 degrees
- Relationship between line slope and the tan function

3B.4. Trigonometric Identities

- Angle sum and difference formulas
- Double angle identities
- Half angle identities
- Solving trigonometric equations

3C. Counting, Probability & Statistics, Number Theory

3C.1. Counting Combinations

- Correcting for overcounting
- Counting combinations: $\binom{n}{r}$

3C.2. More Probability

- Calculating probabilities using combinations & other counting techniques
- Conditional probability

3C.3. Pascal's Triangle and the Binomial Theorem

- Pascal's Triangle
- The Binomial Theorem: coefficients of $(x + y)^n$

3C.4. Calculating Digits, especially the Last Digit

- Finding the last digit of a sum, product, or power of integers
- Counting trailing zeros of products of integers

Meet 4 Topics Breakdown

4A. Algebra

4A.1. Optimization Problems

- Minimum or maximum value of quadratic expressions (e.g. by completing the square)
- The Arithmetic-Geometric mean inequality

4A.2. Roots of Polynomial Equations

- Vieta's formula for the sum of roots of a polynomial
- Vieta's formula for the product of roots of a polynomial
- Vieta's formula for relationships between roots and other coefficients of a polynomial

4A.3. Geometric Sequences and Series

- Geometric sequence definition
- Finding the common ratio
- Finding the n th term
- Geometric series definition
- Calculating the sum of a geometric series
- Geometric sequence & series problem solving

4A.4. More Logarithms & Exponents

- The change of base formula: $\log_a b = \frac{\log_c b}{\log_c a}$
- Solving exponential equations
- Solving logarithmic equations

4B. Geometry & Trigonometry

4B.1. Area, Perimeter, and Angles in Circles

- Sectors
- Circular segments
- Central & inscribed angles
- The Inscribed Angle Theorem

4B.2. Lengths Involving Circles

- Internal and External tangent lines
- Power of a Point
- Cyclic quadrilaterals
- Angle relationships in cyclic quadrilaterals (e.g. opposite angles)
- Problem solving in cyclic quadrilaterals (e.g. Ptolemy's Theorem, Brahmagupta's Formula)

4B.3. Analytic Geometry of Circles

- Circle equations
- Intersections between circles

4B.4. Law of Sines & Law of Cosines

- The Law of Sines
- The Law of Cosines
- Finding lengths and angles in triangles and other geometric figures using these laws

4C. Counting, Probability & Statistics, Number Theory

4C.1. More Counting & Probability

- Counting integer solutions to $x_1 + x_2 + \dots + x_n = k$ ("sticks and stones"), including positive and non-negative cases
- Counting more than two groups (multinomials)
- Counting paths in a grid
- Counting & probability problems using all topics to date

4C.2. Expected Value

- Definition based on individual probabilities
- Expected value problem solving

4C.3. Remainders & Modular Arithmetic

- Calculating remainders
- Modular arithmetic notation
- Remainders of sums, products, and powers of integers

4C.4. Finding Integer Solutions

- Finding integer solutions by factoring & casework
- Finding integer solutions by completing the rectangle ("Simon's Favorite Factoring Trick")

Meet 5 Topics Breakdown

5A. Algebra

5A.1. Non-Linear Equations and Systems of Equations

- Solving radical equations
- Identifying extraneous roots
- Solving more complex equations or systems of equations (combining any techniques used to date)

5A.2. Functional Equations

- Solving for a function based on given properties of the function
- Finding function value for a specific input given properties of the function
- Domain and Range; Function Composition; Inverse Functions
- Functional Operations (Invented operators)

5A.3. Sums of Powers of Integers

- Formula for $1 + 2 + \dots + n$
- Formula for $1^2 + 2^2 + \dots + n^2$
- Formula for $1^3 + 2^3 + \dots + n^3$
- Finding related sums using these formulae

5A.4. General Sequences & Series

- The Fibonacci sequence
- Recursively defined sequences
- Telescoping sums
- General sequence & series problem solving

5B. Geometry & Trigonometry

5B.1. 3-Dimensional Geometry

- Triangular and polygonal prisms & pyramids, cylinders, cones, spheres
- Calculating volume and surface area

5B.2. Geometry Problem Solving

- Problem solving using all Geometry topics
- Could include any topics from subtopics 1 and 2 of previous B events
- Specifically topics 1B.1, 1B.2, 2B.1, 2B.2, 3B.1, 3B.2, 4B.1, 4B.2

5B.3. Analytic Geometry of Conic Sections

- Axis of symmetry
- Focus-directrix definition of a parabola
- Parabola equations
- Focus-focus definition of an ellipse
- Ellipse equations
- Focus-focus definition of a hyperbola
- Hyperbola equations

5B.4. Geometry of Complex Numbers

- The Complex Plane
- Modulus of Complex Numbers
- Distance in the Complex Plane
- Polar representation of a complex number, relationship to trigonometry
- De Moivre's Theorem

5C. Counting, Probability & Statistics, Number Theory

5C.1. Divisor Arithmetic

- Counting divisors of a number
- Sum of divisors of a number

5C.2. Venn Diagrams & the Principle of Inclusion-Exclusion

- Counting with Venn diagrams (2 or 3 sets)
- Counting using the principle of inclusion-exclusion (any number of sets)

5C.3. Geometric Probability

- Calculating probability as a fractional area

5C.4. More Integer Solutions

- More general diophantine equations
- Solving modular equations and systems